1. The equation of state for \( n \) moles of an ideal gas contained in a volume \( V \), and at pressure \( P \) and temperature \( T \), is given by

\[ PV = nRT. \]

(a) Using the fact that the internal energy \( E \) of an ideal gas is a state function of the temperature only, \( E = E(T) \), show that from the first law of thermodynamics that the difference between the heat capacities at constant pressure and at constant volume of \( n \) moles of an ideal gas is given by

\[ C_P - C_V = nR. \]

(b) Hence show that the heat transferred in an infinitesimal quasistatic process involving \( n \) moles of an ideal gas is given by

\[ \bar{d}Q = \frac{1}{nR}(C_V dP + C_P dV). \]

(c) Hence show that for a quasistatic adiabatic process involving an ideal gas, for which the heat capacities are constants (i.e., independent of temperature),

\[ PV^\gamma = K, \]

where \( K \) is a constant, and \( \gamma \equiv C_P/C_V. \)

2. As we proved in Question 1(c) above, during a quasistatic adiabatic compression of an ideal gas, the pressure \( P \) and volume \( V \) are related by the equation

\[ PV^\gamma = K. \]

(a) Show that the work done on the gas during a quasistatic adiabatic compression from a state \((P_i, V_i, T_i)\) to state \((P_f, V_f, T_f)\) is given by

\[ W = \frac{P_f V_f - P_i V_i}{\gamma - 1} = \frac{P_i V_i [(V_i/V_f)^{\gamma-1} - 1]}{\gamma - 1}. \]
(b) By making use of the result in Question 1(a) above, show that this equation for $W$ can also be written as

$$W = C_V(T_f - T_i).$$

Show how this latter expression also follows directly from conservation of energy and the fact that $C_V$ is constant (i.e., independent of temperature) for an ideal gas.

(c) If the volume of the gas is halved during the compression, show that

$$T_f = 2^{\gamma-1}T_i.$$