1. (a) The time-dependent Hamiltonian $\hat{H} = \hat{H}_0 + \hat{V} (t)$ with
\[
\hat{H}_0 = \frac{eB_0}{m} \hat{S}_z, \quad \hat{V} (t) = \frac{eB_1}{m} \left( \hat{S}_x \cos \omega t + \hat{S}_y \sin \omega t \right)
\]
describes a spin $S = 1/2$ system which is subject to the static magnetic field $(0, 0, B_0)$ and the rotating magnetic field $(B_1 \cos \omega t, B_1 \sin \omega t, 0)$. $B_0$ and $B_1$ are uniform throughout space. Initially, at time $t = 0$, the spin of the system is pointing in the direction of the negative $z$-axis. Assuming $B_1$ is weak and taking $\hat{V} (t)$ as perturbation, calculate the probability that at time $t > 0$ the spin points in the positive $z$-axis.

(b) Discuss at what value of $\omega$, the perturbation theory breaks down for sufficiently large $t$, however weak $B_1$. Suggest a criterion in $t$ for the perturbation theory to hold.

2. The spin interaction energy of positronium in a magnetic field can be written as a $(4 \times 4)$ matrix Hamiltonian
\[
\hat{H} = \begin{pmatrix}
\varepsilon_1 & v & 0 & 0 \\
v & \varepsilon_2 & 0 & 0 \\
0 & 0 & \varepsilon_3 & 0 \\
0 & 0 & 0 & \varepsilon_4
\end{pmatrix}.
\]
Calculate all eigenvalues of $\hat{H}$ exactly. Compare the perturbation results of Exercise 3.2 for the energies with the corresponding exact energies under the conditions that $\varepsilon_1 < \varepsilon_2$ and $v \ll \varepsilon_2 - \varepsilon_1$.

3. In $\text{H}_2^+$ problem, using molecular orbitals as linear combination of two hydrogen atomic ground-state orbitals $\phi_a$ and $\phi_b$, i.e., $\psi = c_a \phi_a + c_b \phi_b$, we derive the matrix equation
\[
\begin{pmatrix}
\alpha - E & \beta - ES \\
\beta - ES & \alpha - E
\end{pmatrix}
\begin{pmatrix}
c_a \\
c_b
\end{pmatrix} = 0,
\]
where three integrals are defined as $\alpha = \langle \phi_a | \hat{H} | \phi_a \rangle$, $\beta = \langle \phi_a | \hat{H} | \phi_b \rangle$ and $S = \langle \phi_a | \phi_b \rangle$.

(a) Determine energy $E$ and molecular wavefunction coefficients $c_a$ and $c_b$ in terms of $\alpha$, $\beta$ and $S$.

(b) The above matrix equation holds for a homonuclear diatomic molecule. Write down similar equation for a heteronuclear molecule. Explain any integral you may use. Determine the energy $E$ in terms of these integrals.
4. (a) Write down the ground-state electron term of H\textsubscript{2} molecule.
(b) Write down the corresponding ground-state wavefunction and explain each notation you use.

5. (a) Write down the ground-state electron configuration of Li\textsubscript{2}, Be\textsubscript{2} and N\textsubscript{2}, using molecule orbitals discussed in class.
(b) By counting the bond order of these configurations, determine which molecules is unstable and which molecule is stable. Derive the ground-state molecular term of the stable molecules.