

You may assume the following formulae without proof:

The partial-wave expansion of a plane wave is

$$e^{ikz} = \sum_l \sqrt{4\pi(2l+1)} i^l j_l(kr) Y_{l0}(\theta),$$

where the regular spherical Bessel functions have the asymptotic forms for large x

$$j_l(x) \rightarrow \frac{\sin(x - l\pi/2)}{x}.$$

The differential cross section for elastic scattering in the centre-of-mass frame is

$$\frac{d\sigma}{d\Omega} = |f(\theta, \phi)|^2$$

where, for a central interaction,

$$f(\theta, \phi) = \frac{1}{k} \sum_l \sqrt{4\pi(2l+1)} e^{i\delta_l} \sin \delta_l Y_{l0}(\theta).$$

Here the Y_{l0} are normalised spherical harmonics,

$$Y_{l0}(\theta) = \sqrt{\frac{2l+1}{4\pi}} P_l(\cos \theta),$$

with

$$P_0(\cos \theta) = 1, \quad P_1(\cos \theta) = \cos \theta, \quad P_2(\cos \theta) = \frac{1}{2}(3 \cos^2 \theta - 1).$$

In terms of the amplitudes η_l for the outgoing waves, the elastic-scattering and reaction cross sections are

$$\sigma_{\text{el}} = \frac{\pi}{k^2} \sum_l (2l+1) |1 - \eta_l|^2, \quad \sigma_{\text{re}} = \frac{\pi}{k^2} \sum_l (2l+1) (1 - |\eta_l|^2).$$

The spherical Bessel functions $j_l(x)$ have maxima at the values of x given in the table below.

| $l = 0$ | $l = 1$ | $l = 2$ | $l = 3$ | $l = 4$ |
|---------|---------|---------|---------|---------|
| 0.00 | 2.08 | 3.34 | 4.51 | 5.65 |
| 4.49 | 5.94 | 7.29 | 8.58 | 9.84 |
| 7.73 | 9.20 | 10.61 | 11.97 | 13.30 |

The energy of the Gamow peak is

$$E_0 = 1.22(Z_a^2 Z_b^2 \mu T_6^2)^{1/3} \text{ keV}, \text{ where } T = T_6 \times 10^6 \text{ K and } \mu \text{ has units of u.}$$

The reaction cross section in the sharp-cut-off model is

$$\sigma_{\text{sco}} = \pi R^2 \left(1 - \frac{E_b}{E}\right).$$

The atomic mass unit is $1 \text{ u} = 931.5 \text{ MeV}/c^2$.