PHYS 40602 RELATIVISTIC QUANTUM PHYSICS

3.2 Feynman diagrams and Feynman rules

In covariant perturbation theory for a relativistic scattering process, each term can be represented by a Feynman diagram with one vertex for each interaction and an external line for each initial or final particle. This diagram provides us with a mnemonic for writing down the corresponding contribution to the invariant amplitude $\mathcal{M}_{fi}$. The diagrams representing the types of processes we shall study in the course can be converted into invariant amplitudes with the help of these rules. For each element of the diagram, write down the following:

Rule 0: a factor of i;

Rule 1: a factor of $-\mathrm{i}\, g$ for each vertex where a charged boson or fermion emits a neutral spin-0 boson; remember to conserve 4-momentum at each vertex;

Rule 2: a factor of

$$\frac{\mathrm{i}}{k^2 - m^2 + \mathrm{i}\epsilon}$$

for each internal boson line carrying 4-momentum k;

Rule 3: the following factors for the external lines:

- 1 for each initial or final boson,
- $u_s(\mathbf{p})$ for each initial fermion,
- $\overline{u}_s(\mathbf{p})$ for each final fermion,
- $\overline{v}_s(\mathbf{p})$ for each initial antifermion,
- $v_s(\mathbf{p})$ for each final antifermion;

multiply together the fermion factors, starting on the right of the expression and following the direction of the charge-flow arrows on the diagram;

Rule 4: symmetrise identical bosons in the final state by drawing the diagram(s) where the final particles are exchanged and adding their contributions to the amplitude;
similarly antisymmetrise identical fermions in the final state by including a minus sign for any diagram where a pair of fermions has been exchanged.

[Rule 0 follows from the definition of the invariant amplitude, Rule 1 from the fact that we are expanding $e^{-\mathrm{i}H_I t}$. Virtual particles created by one interaction and destroyed at another are described by the Feynman propagators of Rule 2. Rule 3 supplies the Fourier transforms of the waves describing the initial and final particles, all with covariant normalisations. The indistinguishability of identical particles is implemented by Rule 4.]

To describe electromagnetic scattering processes, we need to extend the rules as follows:

Rule 1: a factor of $-\mathrm{i}\, q\gamma_\mu$ for each vertex where a fermion of charge q emits or absorbs a photon with polarisation μ;
a factor of $-\mathrm{i}\, q(p_{1\mu} + p_{2\mu})$ for each similar vertex involving a charged boson with initial 4-momentum p_1 and final 4-momentum p_2;

Rule 2: a factor of

$$\frac{-\mathrm{i}\, g^{\mu\nu}}{k^2 + \mathrm{i}\epsilon}$$

for each internal photon line carrying 4-momentum k.

Further extensions of these rules allow us to describe processes with initial or final photons, processes with virtual fermions, and processes which involve quantum mechanical fluctuations of the fields (represented by diagrams with closed loops). Complete sets of Feynman rules for Quantum Electrodynamics can be found in any textbook on quantum field theory. These rules can also be extended to describe theories with more complicated interactions between the fields, such as Quantum Chromodynamics or Electroweak Theory.

Mike Birse (April 2010)