PHYS 30101 APPLICATIONS OF QUANTUM PHYSICS SOLUTIONS 1

Only "bottom-line" answers are given for the short questions 1 to 6.

1. (a) $\lambda = 630$ nm

   (b) $\lambda = 0.9$ nm

2. (a) $p = \hbar k$ (every time)

   (b) $p = +\hbar k$ and $-\hbar k$, with equal probabilities

3. (a) Positive x

   (b) Negative y

   [Think about the points where the phase, e.g. $kx - \omega t$, is constant. How do these change with time?]

4. Eigenfunctions: $e^{im\phi}$; eigenvalues: $m\hbar$ where m is an integer.

5. (a) $E_n = \dfrac{n^2\pi^2\hbar^2}{2ma^2}$ with $n \geq 1$

   (b) $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$ where $\omega = \sqrt{k/m}$, with $n \geq 0$

   (c) $E_n = -\dfrac{E_R}{n^2}$ where $E_R = \dfrac{m_e}{2}\left(\dfrac{e^2}{4\pi\epsilon_0\hbar}\right)^2$, with $n = n_r + l + 1$, $n_r \geq 0$ and $l \geq 0$

6. $P(x,t) = |\psi(x)|^2$

7. (a) Inside the well, $0 < x < a$, the eigenfunctions satisfy the free Schrödinger equation,

   $$-\frac{\hbar^2}{2M}\frac{d^2\psi}{dx^2} = E\,\psi.$$

   The boundary conditions they satisfy at the walls are

   $$\psi(0) = \psi(a) = 0.$$

   If you substitute in the function given in the question, you should find that it satisfies the TISE and both boundary conditions, but only if the energy E is one of the eigenvalues given. [You weren't asked to solve the eigenvalue problem from first principles, so you don't need to start with the general solution $\psi(x) = A\cos(kx) + B\sin(kx)$.]

   (b) These look just like the first three normal modes of a guitar string. See Rae, Figure 2.1.

(c) A wave function is normalised if the total probability for finding the particle somewhere adds up to 1:

$$\int_{-\infty}^{+\infty} |\psi_n(x)|^2 \, \mathrm{d}x = 1.$$

For the eigenfunctions in this problem, we can use either the trig identity $\sin^2 X = \frac{1}{2}[1 - \cos(2X)]$ or the fact that $\sin^2$ averages to $1/2$ over each half wavelength. Whichever you choose should give you

$$B^2 \int_0^a \left(\sin \frac{n\pi x}{a}\right)^2 \mathrm{d}x = B^2 \frac{1}{2} a = 1.$$

(d) Adding up the probabilities $P(x)\mathrm{d}x$ for finding the particle in the region $0 < x < a/2$, we get

$$P[0 < x < a/2] = \int_0^{a/2} |\psi_n(x)|^2 \, \mathrm{d}x = B^2 \frac{1}{2} \frac{a}{2} = \frac{1}{2}.$$

(e) The expectation value of x is defined by

$$\langle x \rangle = \int_{-\infty}^{+\infty} x |\psi_n(x)|^2 \, \mathrm{d}x.$$

The integral here can be done with the aid of the trig identity above and integration by parts.

This expectation value is the average value for the position of a particle described by $\psi_n(x)$ and the eigenfunctions all give probability distributions $|\psi_n(x)|^2$ that are symmetric about $x = a/2$. Hence it isn't surprising that the integral leads to $\langle x \rangle = a/2$.

(f) This integral can be done using the identity $\sin X \sin Y = \frac{1}{2}[\cos(X - Y) - \cos(X + Y)]$.