PHYS 30121 NUCLEAR AND PARTICLE PHYSICS EXAMPLES 4

Questions 1 and 2 are on radioactive decay (and the concept of a cross section). Questions 3 and 4 use Q values to deduce features of particular β and α decays. Questions 5, 6 and 7 apply the simple shell model to the spins and parities of nuclear states.

1. Natural bismuth consists entirely of ^{209}Bi which has a half-life $\tau \simeq 1.9 \times 10^{19}$ years. What mass of bismuth would be needed in order to observe one decay per day on average?
   [Here and in question 2, you may (for once) use SI units, where 1 u $= 1.66\times10^{-27}$ kg.]

2. (a) A radioactive isotope has a decay constant λ, and it is produced at a constant rate R. Write down a differential equation for the number $N(t)$ of atoms of the isotope at time t. Assuming that there are no atoms of the isotope present at $t = 0$, show that
   $$N(t) = \frac{R}{\lambda}\left(1 - \mathrm{e}^{-\lambda t}\right).$$
   Hence find the activity (number of decays per unit time) at time t. Comment on the behaviour of the activity at large times.

   (b) ^{60}Co is produced in a reactor by bombarding natural cobalt (100% ^{59}Co) with an intense flux of low-energy neutrons. The cross section for capture of a low-energy neutron by ^{59}Co is $\sigma = 37$ b. Calculate the production rate of ^{60}Co for a 100 g block of cobalt placed in a reactor that generates a neutron flux $J_n = 10^{13}$ cm^{-2}s^{-1}.

   (c) The cobalt block in part (b) is to be used to produce a ^{60}Co radiation source with an activity of 100 TBq. For how long must it be placed in the reactor? [The half-life of ^{60}Co is 5.27 years. You may ignore capture of neutrons by ^{60}Co, which has a much smaller cross section.]

3. The isobars with $A = 100$ closest to the line of stability have the following mass excesses.

Isobar	Δ (MeV)
$^{100}_{40}$Zr	-76.60
$^{100}_{41}$Nb	-79.94
$^{100}_{42}$Mo	-86.18
$^{100}_{43}$Tc	-86.02
$^{100}_{44}$Ru	-89.22
$^{100}_{45}$Rh	-85.58
$^{100}_{46}$Pd	-85.23
$^{100}_{47}$Ag	-78.15

   Which weak decays (such as $\beta^{\pm}$ or electron capture) do you expect for each of these nuclei? Which of these nuclei could be stable?

4. Lead-208 ($^{208}_{82}$Pb) is currently the heaviest "stable" nucleus, in the sense that it has never been observed to decay. Show that, in principle, it is able to decay to $^{204}_{80}$Hg by emitting an α particle. (The relevant mass excesses are $\Delta(^{208}_{82}\text{Pb}) = -21748$ keV, $\Delta(^{204}_{80}\text{Hg}) = -24690$ keV, and $\Delta(^{4}\text{He}) = 2425$ keV.)

   For α decay of ^{208}Pb, find the Gamov energy,

$$E_G = 2(\pi Z_1 Z_2 \alpha)^2 M_r c^2,$$

   and the tunnelling factor,

$$T = \exp\left(-\sqrt{\frac{E_G}{Q}}\right).$$

   [Your calculator will almost certainly give up on this, so you may need to use $\log_{10} x = \ln(x)/\ln(10)$.]

   Hence use the half-life of ^{209}Bi from question 1 to estimate the lifetime of ^{208}Pb. [In lectures we saw that α decay of ^{209}Bi has $Q = 3.14$ MeV and $E_G = 101$ GeV.]

5. Consider the subshell corresponding to the $1f_{7/2}$ orbital in a nucleus. Write down the value of j and the possible values of m_j for the states in this subshell. What is the maximum number of neutrons that can occupy this subshell?

   For an $1f_{7/2}$ subshell that is completely filled with neutrons, what is the total magnetic quantum number M? What is the corresponding value for J (the quantum number giving the magnitude of the total angular momentum)? What is the overall parity of this system?

   Use this result to predict the spin and parity of the ground state of $^{48}_{20}$Ca, given that the ground state of $^{40}_{20}$Ca has $J^P = 0^+$.

6. The order of the lowest levels in the simple spherical shell model is:

$$1s_{1/2} \;\Big|\; 1p_{3/2}\; 1p_{1/2} \;\Big|\; 1d_{5/2}\; 2s_{1/2}\; 1d_{3/2} \;\Big|\; 1f_{7/2} \;\Big|\; 2p_{3/2}\; 1f_{5/2}\; 2p_{1/2}\; 1g_{9/2} \;\Big|,$$

   where the vertical lines indicate the positions of the larger energy gaps.

   (a) Write down the shell-model configurations of protons and neutrons in the ground state of $^{48}_{20}$Ca. Explain why $^{48}_{20}$Ca is a particularly stable isotope, despite having a rather large excess of neutrons over protons compared with other stable nuclei in its region of A (which typically have $N - Z \simeq 4$).

   (b) Use the simple shell model to predict the spins and parities of the ground states of: i) $^{49}_{20}$Ca, ii) $^{49}_{21}$Sc, iii) $^{47}_{20}$Ca.

   (c) The spectrum of $^{49}_{20}$Ca contains excited states with $J^P = \frac{1}{2}^-$ and $\frac{5}{2}^-$ with energies 2.02 MeV and 3.59 MeV. Suggest interpretations for these states in the shell model.

(d) Predict the spins and parities of the ground states of ${}^{50}_{20}\text{Ca}$ and ${}^{46}_{20}\text{Ca}$.

(e) The ground state of ${}^{50}_{21}\text{Sc}$ has $J^P = 5^+$. Give a shell-model interpretation of this state, and suggest low-lying states you would expect to see in the excitation spectrum of ${}^{50}_{21}\text{Sc}$.

7. The spin-orbit interaction in nuclei has the form

$$\widehat{H}_{\text{so}} = -\frac{\mathcal{E}}{\hbar^2}\,\widehat{\mathbf{L}}\cdot\widehat{\mathbf{S}},$$

where $\mathcal{E}$ is a positive constant. This interaction splits a single-particle orbital with orbital angular momentum l into two, with $j = l + \frac{1}{2}$ and $j = l - \frac{1}{2}$.

(a) Show that the splitting between the two levels can be written as

$$\Delta E = \mathcal{E}\,\frac{2l+1}{2}.$$

(b) Use the information on the spectrum of ${}^{49}_{20}\text{Ca}$ in question 6(c) to deduce a value of $\mathcal{E}$ for the $2p$ orbital for neutrons in this nucleus.

(c) Consider a nucleus where both the $j = l + \frac{1}{2}$ and $j = l - \frac{1}{2}$ subshells are fully occupied. Show that the total contribution of the spin-orbit interaction to the energy of the nucleus is zero.