Questions 1 to 3 are on angular momentum and the discrete symmetries P and C. In some cases they also involve the exchange symmetries of bosons or fermions, or the quark model for hadrons. Questions 4 and 5 apply these ideas to weak interactions. Question 6 is on isospin multiplets, in a nuclear physics context.

1. The ground state of ^{7}Li consists of one proton and two neutrons bound in p-wave orbitals to a ^{4}He core. The ^{4}He core has spin-parity $J^P = 0^+$. What is the parity of the ^{7}Li ground state?

2. The quarks in an excited meson have relative orbital angular momentum $L = 1$ and total spin $S = 1$. Find the possible values of the total internal angular momentum ("spin") J of such a particle. Deduce the particle's parity and C parity and list its possible J^{PC} values. Try to identify candidates in the listings of the Particle Data Group. [*Hint*: look at the mass region around 1 GeV/c^2 and just above.]

3. The π^0 meson is observed to decay to two photons whose polarisation (i.e. spin) state is antisymmetric. The C parity of the photon is $C = -1$.
   (a) Show that the intrinsic parity of the pion must be odd.
   (b) Determine the C parity of the π^0.
   (c) Very rarely a π^0 meson decays to an e^+e^- pair with relative angular momentum $L_r = 0$ and total spin $S = 0$. Find the parity and C parity of this final state, and use your answers to parts (a) and (b) to check that this could be an electromagnetic decay.

4. The ρ^0 meson has spin $J = 1$. It decays to $\pi^+\pi^-$ pairs which have (relative) orbital angular momentum $L = 1$. It has a width in energy of $\Gamma = 149$ MeV.
   (a) Which interaction do you expect to be responsible for this decay? Check that the conservation of baryon number B and isospin I_3 are consistent with your answer.
   (b) Deduce the parity and C parity of the ρ^0.
   (c) Can a ρ^0 meson decay strongly to a pair of π^0s?
   (d) Can a ρ^0 meson decay electromagnetically to an e^+e^- pair with $L = 0$? Draw a diagram representing this process.
   (e) Show that the J^{PC} values you have found for the ρ^0 are compatible with it being an s-wave state in the quark model.

5. Show that the combined CPT "reflection" transforms a left-handed particle into a right-handed antiparticle with the same momentum. Given that we observe left-handed fermions interacting weakly, what does the CPT theorem tell us about right-handed antifermions, and about right-handed fermions?

6. In 1958, Goldhaber and coworkers determined the helicity of the neutrinos created by the weak interaction. They used ^{152}Eu which decays by "capturing" an electron from an s-wave atomic orbital to produce an excited state of ^{132}Sm and a neutrino. The ^{132}Sm then decays to its ground state by emitting a photon:

$$e^- + {}^{132}\text{Eu} \to {}^{132}\text{Sm}^* \to {}^{132}\text{Sm} + \gamma + \nu_e.$$

The ground states of both nuclei have $J = 0$. By observing the recoiling ^{132}Sm, the experimenters were able to select decays where the photon and neutrino were emitted back-to-back. They were able to determine the helicity (circular polarisation) of the photons from their absorption in magnetised iron.

Choosing your z axis to point along the direction of the photon in these back-to-back decays, show that conservation of J_z allows two possibilities for the helicities of the photon and neutrino (depending on the spin of the original electron). Verify that these can be related by a parity reflection. What was the helicity of the photons that were observed?

7. Consider two nucleons moving in an s-wave. What values of the total spin S are allowed for each of the three charge states (pp, np and nn)? Assign isospin quantum numbers (I_3 and I) to your states.