

Questions 1 and 2 are simple ones on masses and lengths, to allow you to practice working with appropriate units. (Do not even think about using SI units for masses and energies here!) You should do these before the first examples class.

Questions 3 to 6 are on basic ideas from lectures 1 to 4, including some revision of aspects of angular momentum and parity that you met briefly in Solid State last year.

A very useful constant:  $\hbar c \simeq 200 \text{ eVnm} = 200 \text{ MeVfm} = 200 \text{ TeVzm}$

The atomic mass unit:  $1 \text{ u} \simeq 931.5 \text{ MeV}/c^2$

1. (a) The top quark has a mass  $m_t = 173 \text{ GeV}/c^2$ . Find an element whose atom has a similar mass to one top quark.

[Tables of atomic masses can be found in Appendix E.3.1 of Martin's book or Appendix C of Krane's. They can also be obtained online from the Atomic Mass Data Centre: <http://amdc.in2p3.fr/web/masseval.html> or from the IAEA's live chart of nuclides:

<https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html>]

- (b) When the LHC restarts, pairs of protons will collide with an energy of 14 TeV. In the unbelievably unlikely event that all the energy in one these collisions were to go to produce pairs of lead nuclei and antinuclei, estimate the number of pairs that would be produced? [Here you may assume that the mass of a lead nucleus is 208 u.]
- (c) The isotope  $^{209}_{83}\text{Bi}$  was long believed to be stable. Using the current best values for the relevant masses from the AMDC:

$$M(^{209}_{83}\text{Bi}) = 208.980399 \text{ u}$$

$$M(^{205}_{81}\text{Tl}) = 204.974428 \text{ u}$$

$$M(^4_2\text{He}) = 4.002603 \text{ u},$$

show that it is in fact unstable and estimate the energy released when an atom of  $^{209}_{83}\text{Bi}$  decays.

2. (a) The newly discovered Higgs boson has a mass  $m_H = 126 \text{ GeV}/c^2$ . Higgs exchange gives rise to a very short-range force between  $W$  bosons. Estimate the range of this force.
- (b) The dilaton is a hypothetical particle that arises in some theories. Exchange of this particle would modify the inverse-square law for the force between two masses but very careful experiments have ruled this out for distances larger than 0.1 mm. What does this imply about the mass of the dilaton (if it exists)?
- (c) The  $Z^0$  boson has a width in energy  $\Gamma = 2.5 \text{ GeV}$ . Estimate its lifetime.

3. Antineutrinos can be detected when they interact with a proton via the weak force to produce a positron and a neutron. Write down a symbolic expression for this process and draw the corresponding lowest-order Feynman diagram. [You should treat the proton and neutron as “elementary” here.]

4. Draw the lowest-order Feynman diagrams (a) for Compton scattering on an electron:

$$\gamma + e^- \rightarrow \gamma + e^-,$$

and (b) for electron-positron scattering,

$$e^- + e^+ \rightarrow e^- + e^+.$$

[*Hint: In each case you should have two diagrams.*]

5. A  $W$  boson has an intrinsic spin  $S = 1$  and is in a state with orbital angular momentum  $L = 1$ . What are the possible values for its total angular momentum  $J$ ? What is the largest possible value for the magnitude of its angular momentum  $|\mathbf{J}|$  (in units of  $\hbar$ )?

6. Show that the spherical harmonic

$$Y_{1,+1}(\theta, \phi) = - \left( \frac{3}{8\pi} \right)^{1/2} \sin \theta e^{i\phi}$$

is odd under parity.

[*Hint: Express  $Y_{1,+1}$  in terms of Cartesian coordinates using  $x = r \sin \theta \cos \phi$ ,  $y = r \sin \theta \sin \phi$ ,  $z = r \cos \theta$ .*]