Lecture 3/4

Angular momentum in quantum mechanics

Eigenvalues (same pattern for $\widehat{\mathbf{L}}$, $\widehat{\mathbf{S}}$ and total $\widehat{\mathbf{J}}$)

$$|\mathbf{J}|^2 = J(J+1)\hbar^2, \qquad J \geq 0$$

$$J_z = M_J\,\hbar, \qquad M_J = -J,\ \ldots,\ +J \quad \text{in steps of 1}$$

[Note: J denotes the quantum number, not the magnitude ($|\mathbf{J}|$)]

Orbital angular momentum: quantum numbers L, M_L must be integer

Intrinsic spin: S, M_S can be half-integers

π, H	$S = 0$
e, ν, N, q	$S = \frac{1}{2}$
g, γ, W, Z	$S = 1$

Addition of angular momenta $\widehat{\mathbf{J}} = \widehat{\mathbf{J}}_1 + \widehat{\mathbf{J}}_2$

possible values of total angular momentum quantum number

$$J = |J_1 - J_2|,\ \ldots,\ J_1 + J_2 \quad \text{in steps of 1} \quad \text{(triangle rule)}$$

Parity

Reflection of all coordinates: $\mathbf{r} \to -\mathbf{r}$

Vectors $\mathbf{r}$, $\mathbf{p}$ odd under parity; scalars $|\mathbf{r}|$, t even
Angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ even ("axial" vector)

Assuming Hamiltonian is even $\to$ parity P conserved
P: multiplicative quantum number

Single particle with orbital angular momentum L:

$$P = (-1)^L$$

Two particles with L_1, L_2: $P = (-1)^{L_1+L_2}$ (multiply)
or with relative angular momentum L_r (ignoring L_{CM}): $P = (-1)^{L_r}$
Photon intrinsic parity: $P = -1$
Fermion and antifermion: opposite intrinsic parities

$$\begin{array}{ll} e^-, \nu, N, q & P = +1 \\ e^+, \bar{\nu}, \bar{N}, \bar{q} & P = -1 \end{array}$$

Internal symmetries

Symmetries under rotations of phases of wave functions

$\rightarrow$ conserved charges

electric charge	Q	absolutely conserved
baryon number	B	strong, EM and weak
		(not in Big Bang or GUTs)
lepton number	L	like B (but ν masses?)
isospin 3rd component	I_3	strong and EM but not weak

Isospin multiplets: $I_3 = -I, \ldots, +I$ in steps of 1

$$\left.\begin{array}{ll} p: & I_3 = +\frac{1}{2} \\ n: & I_3 = -\frac{1}{2} \end{array}\right\} \text{magnitude } I = \frac{1}{2} \text{ (like spin-}\tfrac{1}{2}\text{)}$$