Lecture 15/16

Shell model

Simplest version

- nucleons move independently in average potential
- potential $\sim$ between harmonic oscillator and square well
  (saturation of short-range forces)
- strong spin-orbit interaction splits orbitals with $j = l \pm \frac{1}{2}$
  larger j has lower energy

$\rightarrow$ level ordering (labels: $n = n_r + 1$, l, j)

$$1s_{1/2} \,\Big|\, 1p_{3/2}\, 1p_{1/2} \,\Big|\, 1d_{5/2}\, 2s_{1/2}\, 1d_{3/2} \,\Big|\, 1f_{7/2} \,\Big|\, 2p_{3/2}\, 1f_{5/2}\, 2p_{1/2}\, 1g_{9/2} \,\Big|$$

- degeneracies of orbitals: $2j + 1$
- completely filled (closed) shells at magic numbers

$$Z,\ N = 2,\ 8,\ 20,\ 28,\ 50,\ 82,\ 126$$

Ground states

Fill shells and subshells in order (Pauli principle)

- filled subshell: $J^P = 0^+$
- valence nucleons (outside filled shell) determine J^P
  alternatively for nearly full shell: valence "holes"
- pairing: even number of valence protons (or neutrons)
  form pairs with $J^P = 0^+$ (all even-even nuclei: $J^P = 0^+$)

Example: ^{17}O, $Z = 8$, $N = 9$

$$
\begin{aligned}
p: &\quad (1s_{1/2})^2\,(1p_{3/2})^4\,(1p_{1/2})^2 \\
n: &\quad (1s_{1/2})^2\,(1p_{3/2})^4\,(1p_{1/2})^2\,(1d_{5/2})^1
\end{aligned}
$$

- one valence neutron in $1d_{5/2} \longrightarrow J^p = \frac{5}{2}^+$

Exited states

Single-particle excitations

- one nucleon excited to higher level in same shell, $E \sim$ few MeV
- example: ^{17}O has $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ states with $E \lesssim 5$ MeV

Mirror nuclei

- swap $Z \leftrightarrow N$; example: ^{17}O and ^{17}F
- very similar spectra (charge symmetry of forces)

Odd-odd nuclei

- unpaired proton with l_p, j_p and neutron with l_n, j_n can give low-energy states with

$$J = |j_p - j_n|, \ldots, j_p + j_n \qquad P = (-1)^{l_p + l_n}$$

Nuclei far from closed shells

- many valence nucleons
- collective rotations and vibrations
- very-low-energy states ($\sim$ few 100 keV)