Lecture 11/12

Nuclear masses

Nucleus specified by

- atomic number Z (protons)
- mass number A (baryon number)
  neutron number $N = A - Z$

Isotopes: same Z, different A; isobars: same A, different Z

Easier to measure atomic masses $M(Z,A)$

Binding energy: $B(Z,A) = \left[Z\,M(^{1}\mathrm{H}) + N\,M_n - M(Z,A)\right]c^2$

Mass excess: $\Delta(Z,A) = M(Z,A) - A\,M_u$ ($M_u = 1\,\mathrm{u} = 931.5$ MeV$/c^2$)

Saturation of nuclear forces: binding energy per nucleon

$$B/A \sim 8\,\mathrm{MeV} \quad \text{for} \quad A \gtrsim 12$$

Q values can usually be calculated from atomic masses only

- α decay: $Q = \left[M(Z, A) - M(Z-2, A-4) - M(^4\mathrm{He})\right] c^2$
- β decay: $Q = \left[M(Z, A) - M(Z+1, A)\right] c^2$
- ignore all electrons provided all particles are constituents of normal atoms(p, n, e^-) or are neutral (γ, ν, $\overline{\nu}$) (electrons already accounted for in atomic masses)
- anything else (e^+, π^-, $\overline{p}$) needs more care!
- changes in electronic binding energies normally negligible

Semi-empirical mass formula

$$M(A,Z) = Z\,M(^1\mathrm{H}) + (A-Z)M_n - a_v\,A + a_s\,A^{2/3} + a_c\,\frac{Z(Z-1)}{A^{1/3}} + a_a\,\frac{(A-2Z)^2}{A} - \delta_p$$

- v: volume (saturation of forces)
- s: surface (fewer neighbours)
- c: Coulomb (repulsion between protons)
- a: asymmetry (Pauli principle for $N \neq Z$)
- p: pairing ($J = 0$ pairs of identical particles)

$$\delta_p = \begin{cases} +a_p\,A^{-1/2} & \text{even-even} \\ 0 & \text{odd } A \\ -a_p\,A^{-1/2} & \text{odd-odd} \end{cases}$$

Typical values, in MeV/c^2

$$a_v = 15.85,\; a_s = 18.34,\; a_c = 0.71,\; a_a = 23.21,\; a_p = 12.0$$