You may use the following data and formula.

The atomic mass unit is $M_u = 931.494$ MeV$/c^2$.

The quantum numbers of the photon are $J^{PC} = 1^{--}$.

The quantum numbers of the pion are $J^{PC} = 0^{-+}$.

The semi-empirical mass formula can be written in the form:

$$M(A,Z) = Z\,M(^1\mathrm{H}) + (A-Z)M_n - a_v\,A + a_s\,A^{2/3} + a_c\,\frac{Z^2}{A^{1/3}} + a_a\,\frac{(A-2Z)^2}{A} - \delta_p\,,$$

where

$$\delta_p = \begin{cases} +a_p\,A^{-1/2} & \text{even-even} \\ 0 & \text{odd } A \\ -a_p\,A^{-1/2} & \text{odd-odd} \end{cases}.$$

1. (a) Draw two lowest-order Feynman diagrams for the process

$$e^- + e^+ \to \nu_e + \overline{\nu}_e.$$

[5 marks]

(b) Atoms of $^{22}_{11}$Na decay to $^{22}_{10}$Ne emitting positrons with a maximum energy of 0.546 MeV. The $^{22}_{10}$Ne is produced in a state with an excitation energy of 1.275 MeV. The mass excess of the $^{22}_{10}$Ne ground state is -8.025 MeV$/c^2$. Find the mass of $^{22}_{11}$Na in atomic mass units. Give your answer to 7 significant figures. (You may neglect nuclear recoil.) [6 marks]

(c) Explain briefly the physical origins of the term $a_a(A-2Z)^2/A$ in the semi-empirical mass formula. [5 marks]

(d) Nuclear β decay arises from the weak interaction by exchange of a virtual W boson. The W-boson mass is $M_W = 80.4$ GeV. Estimate the range of the weak force and compare your result with the size of a nucleus. [4 marks]

(e) A particle X is produced by the strong-interaction process

$$K^- + p \to K^+ + X.$$

Deduce the charge, baryon number, third component of isospin and strangeness of this particle, and hence state its quark constituents. [5 marks]

2. (a) Describe what is meant by parity reflection (P), charge conjugation (C), and time reversal (T). State the effect of each of these on a particle with momentum $\mathbf{p}$ and angular momentum $\mathbf{J}$. For each of the known fundamental forces, indicate which of these symmetries and their combination CPT are respected. [9 marks]

   (b) The ψ was the first meson to be discovered that consists of a c quark and a $\overline{c}$ antiquark. It was seen very strongly in e^+e^- annihilation reactions.

      i. Draw a lowest-order quark-flow diagram representing production of a ψ meson in e^+e^- annihilation. [2 marks]

      ii. Explain why this lowest-order process implies that the ψ meson has quantum numbers $J^{PC} = 1^{--}$. Show that these quantum numbers are consistent with a $c\overline{c}$ pair in its orbital ground state. [8 marks]

      iii. Explain why the ψ meson is observed to decay into three photons but not into two photons. Explain also why it does not decay into one photon. [6 marks]

3. The order of the lowest levels in the simple spherical shell model of nuclei is:

$$1s_{1/2} \,\Big|\, 1p_{3/2}\ 1p_{1/2} \,\Big|\, 1d_{5/2}\ 2s_{1/2}\ 1d_{3/2} \,\Big|\, 1f_{7/2} \,\Big|\, 2p_{3/2}\ 1f_{5/2}\ 2p_{1/2}\ 1g_{9/2} \,\Big|,$$

where the vertical lines indicate the larger energy gaps.

(a) Describe briefly the physical significance of "magic numbers" in nuclear physics. Use the above level ordering to find the first three of these numbers. [5 marks]

(b) Explain briefly why the next observed magic number is 28, and not 40 as the three-dimensional harmonic oscillator would predict. [5 marks]

(c) Use the pattern of levels given above to predict the spin and parity for the ground state and the first two excited states of $^{57}_{28}$Ni. [5 marks]

(d) Explain why the nuclei $^{57}_{28}$Ni and $^{57}_{29}$Cu are described as a "mirror pair", commenting on how you expect their low-lying excitation energies to compare. [4 marks]

(e) The separation energy for the least-bound odd nucleon is 10.25 MeV for $^{57}_{28}$Ni and 0.69 MeV for $^{57}_{29}$Cu. Give a reason for this energy difference, supporting your answer with a quantitative estimate. [6 marks]

4. (a) Describe how the simple three-flavour quark model leads to the grouping of the lightest baryons into an octet with $J^P = \frac{1}{2}^+$ and a decuplet with $J^P = \frac{3}{2}^+$. Your answer should explain the role of the "colour" quantum number of quarks and it should include the weight diagrams representing these multiplets, with the proton, neutron and Ω^- labelled. [You do not need to label all the other particles in the multiplets.] [13 marks]

(b) State the approximate mass differences between the particles in a multiplet. Estimate the mass of the Ω^-. [4 marks]

(c) The set of baryons consisting of one charm quark and two light quarks (u, d or s) also includes a multiplet with $J^P = \frac{3}{2}^+$. Write down the number of particles in this multiplet and draw the weight diagram representing it.

The css member of this multiplet, the $\Omega_c(2770)$, has a mass that is 70 MeV/c^2 greater than that of the $J^P = \frac{1}{2}^+$ css ground state. State how you expect the $\Omega_c(2770)$ to decay, justifying your answer, and estimate its lifetime. [You may use the fact that the masses of the π and K mesons are about 140 MeV/c^2 and 495 MeV/c^2, respectively.] [8 marks]

NUMERICAL AND BOTTOMLINE ANSWERS

1. (a) No numerical answer

   (b) 21.99444 u

   (c) No numerical answer

   (d) $\sim 2 \times 10^{-3}$ fm

   (e) $Q = -1$, $B = +1$, $I_3 = -\frac{1}{2}$, $S = -2$

   dss

2. No numerical answers

3. (a) 2, 8, 20

   (b) No numerical answer

   (c) Ground state: $J^P = \frac{3}{2}^-$; excited states: $J^P = \frac{5}{2}^-, \frac{1}{2}^-$

   (d) No numerical answer

   (e) Coulomb energy $E_c \simeq \frac{Z\alpha\hbar c}{R} \simeq 10$ MeV

4. (a) No numerical answer

   (b) $M_\Omega \simeq M_N + 3\Delta M_s + E_{spin}$ with $\Delta M_s \simeq 150$ MeV and $E_{spin} \simeq 300$ MeV

   (c) 6 states

   (d) No numerical answer

   EM decay: $\tau \simeq 10^{-20}$ to 10^{-16} s