You may use the following data and formula.

The atomic mass unit is $M_u = 931.494\ \mathrm{MeV}/c^2$.

The quantum numbers of the photon are $J^{PC} = 1^{--}$.

The quantum numbers of the pion are $J^{PC} = 0^{-+}$.

The semi-empirical mass formula can be written in the form:

$$\begin{aligned} M(A,Z) &= Z\,M(^1\mathrm{H}) + (A-Z)M_n - a_v\,A + a_s\,A^{2/3} + a_c\,\frac{Z^2}{A^{1/3}} \\ &\quad + a_a\,\frac{(A-2Z)^2}{A} - \delta_p\,, \end{aligned}$$

where

$$\delta_p = \begin{cases} +a_p\,A^{-1/2} & \text{even-even} \\ 0 & \text{odd } A \\ -a_p\,A^{-1/2} & \text{odd-odd} \end{cases}.$$

1. (a) Draw two lowest-order Feynman diagrams for scattering of an electron antineutrino by an electron. [5 marks]

   (b) Write down an expression for the dependence of nuclear radii on mass number A and give a brief explanation for its form. [4 marks]

   (c) The order of the lowest levels in the simple spherical shell model is:

   $$1s_{1/2} \;\Big|\; 1p_{3/2}\; 1p_{1/2} \;\Big|\; 1d_{5/2},$$

   where the vertical lines indicate the larger energy gaps. Use this to predict the ground-state spin and parity of $^{15}_{7}\mathrm{N}$. [5 marks]

   (d) Draw a quark-flow diagram for the decay of a Σ^+ strange baryon into a proton and a π^0 meson. Give an estimate for the lifetime of the Σ^+. [5 marks]

   (e) The $f^0(980)$ meson has quantum numbers $J^{PC} = 0^{++}$. Deduce the relative orbital angular momentum L and total spin S of its quarks. [6 marks]

2. (a) The weak interaction is often described as "left handed". Describe briefly what feature of the interaction this refers to. Explain why this implies that the weak interaction violates the symmetries of parity, P, and charge conjugation, C, but it can respect the combined symmetry, CP. [7 marks]

(b) A negatively-charged muon decays into an electron, a neutrino and an antineutrino. Draw the lowest-order Feynman diagram for this process, indicating the flavours of the neutrino and antineutrino. [6 marks]

(c) Other decay modes of the μ^- that have been suggested are the following:

i. $\mu^- \to e^- + \bar{\nu}_e + \bar{\nu}_\mu$

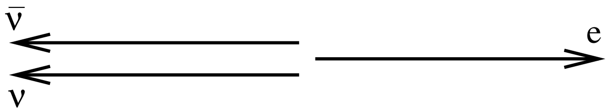
ii. $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu + \gamma$

iii. $\mu^- \to e^- + e^+ + e^-$

iv. $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu + \bar{\nu}_e + \nu_e$

State whether each of these might have been observed, giving reasons. [5 marks]

(d) A high-momentum electron from the decay of a muon is emitted in the opposite direction to the neutrino and antineutrino. These can be assumed to have parallel momenta, as shown in the diagram below.



Draw a similar diagram indicating the directions of the spins of the leptons as well as their momenta. Use conservation of angular momentum to deduce the direction of the spin of the original muon. You may assume that the final leptons have no orbital angular momenta. [5 marks]

(e) Indicate how the result of part (d) might be useful in measuring the magnetic moment of the muon. [2 marks]

3. (a) On a plot of atomic number, Z, against neutron number, N, sketch the line describing the most stable nuclei. Using the semi-empirical mass formula, explain briefly the shape of your curve. Explain also why α decays of the heaviest naturally-occurring nuclei produce daughter nuclei that are unstable against β decay. [10 marks]

(b) The atomic mass excesses of ^{4}He, $^{230}_{90}$Th and $^{234}_{92}$U are: 2425 keV, 30864 keV and 38147 keV. Calculate the energy released, Q, in the α decay of $^{234}_{92}$U. [3 marks]

(c) For α decay of $^{234}_{92}$U, find the Gamow energy,

$$E_G = 2(\pi Z_1 Z_2 \alpha)^2 M_r c^2,$$

and hence the tunnelling factor,

$$T = \exp\left(-\sqrt{\frac{E_G}{Q}}\right).$$

Here Z_1 and Z_2 are the atomic numbers of the daughter nucleus and the α particle, respectively, and M_r is their reduced mass. [4 marks]

(d) $^{238}_{92}$U decays by α decay with a half-life of 4.5×10^9 years and a Q-value of 4270 keV. Use this information to estimate the half-life of $^{234}_{92}$U. [You may take the Gamow energies for ^{234}U and ^{238}U to be equal.] [4 marks]

(e) Estimate the fraction remaining now of the ^{234}U that was present at the formation of the solar system, about 5×10^9 years ago. Explain why natural uranium, which consists dominantly of ^{238}U, contains about 0.005% of ^{234}U. [4 marks]

4. (a) In a simple quark model, the quarks in the lowest-mass mesons are in the ground state of orbital motion. Explain briefly how the three-flavour quark model can lead to a nonet of these mesons with $J = 1$. You do not need to identify the names of these mesons. [5 marks]

   (b) Use the isospins of the quarks to group these mesons into isospin multiplets. [6 marks]

   (c) The non-strange members of this nonet are the ρ^-, ρ^0, ρ^+, ω^0 and ϕ^0 mesons. Their masses are $m_\rho \simeq 770$ MeV$/c^2$, $m_\omega \simeq 780$ MeV$/c^2$ and $m_\phi \simeq 1020$ MeV$/c^2$. Deduce the quark content of each of these mesons. [5 marks]

   (d) Draw a quark-flow diagram for the most likely decay channel of the ϕ^0 meson. [The masses of the lightest mesons are $m_\pi \simeq 140$ MeV$/c^2$ and $m_K \simeq 495$ MeV$/c^2$.] [5 marks]

   (e) The energy width of the ϕ^0 meson is observed to be $\Gamma \simeq 4$ MeV. Find its lifetime and comment on whether this is consistent with the force responsible for the decay in your answer to part (d). [4 marks]

NUMERICAL AND BOTTOMLINE ANSWERS

1. (a) No numerical answer

   (b) No numerical answer

   (c) $J^P = \frac{1}{2}^-$

   (d) No numerical answer

   (e) $L = S = 1$

2. (a) No numerical answer

   (b) No numerical answer

   (c) (i) Forbidden; (ii) allowed; (iii) forbidden (or allowed only as a very rare process though mixing of virtual neutrinos); (iv) allowed (but unlikely to be seen in practice as it involves two weak interactions)

   (d) Opposite direction to momentum of electron

   (e) No numerical answer

3. (a) No numerical answer

   (b) $Q = 4.86$ MeV

   (c) $E_G = 1.25 \times 10^5$ MeV
   $T \simeq 2 \times 10^{-70}$

   (d) $\tau \simeq 10^5$ years

   (e) No numerical answer

4. (a) No numerical answer

   (b) Doublets: $(u\bar{s},\ d\bar{s})$; $(s\bar{d},\ s\bar{u})$
   Triplet: ($u\bar{d}$, $u\bar{u}$ and $d\bar{d}$ superposition, $d\bar{u}$)
   Singlets: $u\bar{u}$ and $d\bar{d}$ superposition; $s\bar{s}$

   (c) (Triplet) ρ^+: $u\bar{d}$, ρ^0: $u\bar{u}$, $d\bar{d}$ (superposition), ρ^-: $d\bar{u}$
   (Singlet) ω^0: $u\bar{u}$, $d\bar{d}$ (superposition)
   (Singlet) ϕ^0: $s\bar{s}$

   (d) No numerical answer

   (e) $\tau \simeq 2 \times 10^{-22}$ s; strong interaction