You may use the following data and formula.

The atomic mass unit is $M_u = 931.494$ MeV/c^2.

The quantum numbers of the photon are $J^{PC} = 1^{--}$.

The quantum numbers of the pion are $J^{PC} = 0^{-+}$.

The semi-empirical mass formula can be written in the form:

$$\begin{aligned} M(A,Z) &= Z\,M(^1\mathrm{H}) + (A-Z)M_n - a_v\,A + a_s\,A^{2/3} + a_c\,\frac{Z^2}{A^{1/3}} \\ &\quad + a_a\,\frac{(A-2Z)^2}{A} - \delta_p\,, \end{aligned}$$

where

$$\delta_p = \begin{cases} +a_p\,A^{-1/2} & \text{even-even} \\ 0 & \text{odd } A \\ -a_p\,A^{-1/2} & \text{odd-odd} \end{cases} .$$

1. (a) Draw all the lowest-order Feynman diagrams that represent Compton scattering on a positron:

$$\gamma + e^+ \to \gamma + e^+.$$

[5 marks]

(b) Explain briefly the physical origin of the terms $-a_v A$ and $+a_s A^{2/3}$ in the semi-empirical mass formula. [5 marks]

(c) The cross section for the reaction $n + {}^{32}\mathrm{S} \to p + {}^{32}\mathrm{P}$ is 60 mb. Calculate the rate at which ^{32}P atoms are produced when 100 g of ^{32}S is exposed to a neutron flux of 10^{14} cm^{-2}s^{-1}. [5 marks]

(d) The η meson has a width of 1.3 keV. State which interaction you expect to be responsible for its decay. Explain your reasoning. [5 marks]

(e) The ϕ meson consists of an s quark and an $\bar{s}$ antiquark in their orbital ground state with a total spin $S = 1$. Deduce the parity and C parity of this particle. [5 marks]

2. The order of the lowest levels in the simple spherical shell model is:

$$1s_{1/2} \Big| \; 1p_{3/2} \; 1p_{1/2} \Big| \; 1d_{5/2} \; 2s_{1/2} \; 1d_{3/2} \Big| \; 1f_{7/2} \Big| \; 2p_{3/2} \; 1f_{5/2} \; 2p_{1/2} \; 1g_{9/2} \Big| ,$$

where the vertical lines indicate the larger energy gaps.

(a) Define the notation used above to label the levels. Outline briefly the origin of this pattern of levels. [10 marks]

(b) Explain why the number 20 is known as a "magic number" in nuclear physics. [4 marks]

(c) Use the pattern of levels given above to predict the spin and parity for the ground state of ${}^{39}_{19}\mathrm{K}$, and for the first two excited states with the same parity. [5 marks]

(d) The ground state of ${}^{40}_{19}\mathrm{K}$ has spin-parity $J^P = 3^-$. Give a shell-model interpretation of this state, and suggest other low-lying states you would expect to see in the excitation spectrum of ${}^{40}_{19}\mathrm{K}$. [6 marks]

3. (a) A nucleus with atomic number Z and mass number A decays by β^+ emission. Define this process, indicating all the particles in the final state. Show that the energy released in this decay is

$$Q = \big[M(Z, A) - M(Z-1, A) - 2m_e\big]c^2,$$

where $M(Z, A)$ and $M(Z-1, A)$ are the atomic masses of the isobars involved and m_e is the electron mass. State clearly any approximations you have used. [7 marks]

(b) The nucleus ${}^{58}\mathrm{Co}$ is observed to decay by β^+ emission. Use the data on mass excesses in the table below to determine the kinetic energy released in this decay. List any other possible weak decays of ${}^{58}\mathrm{Co}$. Indicate, giving reasons, which of the isobars in the table could be absolutely stable and which might have a very long lifetime ($> 10^{20}$ years). [6 marks]

Isobar	Δ (MeV)
${}^{58}_{25}\mathrm{Mn}$	-55.83
${}^{58}_{26}\mathrm{Fe}$	-62.15
${}^{58}_{27}\mathrm{Co}$	-59.85
${}^{58}_{28}\mathrm{Ni}$	-60.23
${}^{58}_{29}\mathrm{Cu}$	-51.67

(c) Shortly after C. S. Wu's experiment on β^- decay of ${}^{60}\mathrm{Co}$, another group studied β^+ decay of ${}^{58}\mathrm{Co}$ in a magnetic field. They found that the positrons are more likely to be emitted along the direction of spins of the ${}^{58}\mathrm{Co}$ nuclei. Explain why this observation shows that parity symmetry is violated in this decay. [6 marks]

(d) In contrast, the electrons from ^{60}Co are emitted more often in the opposite direction to the nuclear spins. Assuming that the light particles in the final states of both decays carry a total spin $S_z = +\hbar$ along the axis of the nuclear spin, use the properties of the weak interaction to explain the difference between the patterns seen in the decays of the two isotopes. [6 marks]

4. (a) Describe briefly the charm quantum number C for hadrons. The Λ_c^+ baryon, the Σ_c^{++} baryon and the D^0 meson all have $C = +1$ and zero strangeness and bottomness. Identify the quark constituents and determine the value of I_3 for each of these hadrons. [8 marks]

(b) What is the isospin quantum number I for the Σ_c^{++} baryon? List any other members of the same isospin multiplet, giving their quark contents. [3 marks]

(c) For each of the following processes state whether it is allowed, and classify the ones that are allowed as strong, electromagnetic or weak. Here $\overline{D}^0$ denotes the antiparticle of the D^0 meson.

i. $p + p \to \Lambda_c^+ + \overline{D}^0 + p$

ii. $p + \overline{p} \to D^0 + \overline{D}^0$

iii. $\Lambda_c^+ \to n + \mu^+ + \nu_\mu$

iv. $\Lambda_c^+ \to \Sigma^+ + \pi^0$

v. $\Lambda_c^+ \to D^0 + e^+ + \gamma$

[8 marks]

(d) Draw quark-level diagrams for the decays $\Lambda_c^+ \to \Lambda^0 + \pi^+$ and $\Lambda_c^+ \to n + K^+$. Explain why you would expect one of these decays to occur at a much higher rate than the other. [6 marks]

NUMERICAL AND BOTTOMLINE ANSWERS

1. (a) No numerical answer
   (b) No numerical answer
   (c) 1.1×10^{13} s^{-1}
   (d) 5×10^{-19} s; EM
   (e) $P = C = -1$
2. (a) No numerical answer
   (b) No numerical answer
   (c) Ground state: $J^P = \frac{3}{2}^+$
   Low-lying single-particle (hole) excitations: $J^P = \frac{1}{2}^+, \frac{5}{2}^+$
   (d) Possible states of $1d_{3/2}$ hole and $1f_{7/2}$ particle: $J^P = 2^-, 3^-, 4^-, 5^-$
3. (a) No numerical answer
   (b) $Q = 1.28$ MeV for β^+ to ^{58}Fe
   Other decays of ^{58}Co: EC to ^{58}Fe, β^- to ^{58}Ni
   Stable: ^{58}Fe
   Lifetime $\gtrsim 10^{20}$ years: ^{58}Ni
   (c) No numerical answer
   (d) No numerical answer
4. (a) Λ_c^+: cud, $I_3 = 0$
   Σ_c^{++}: cuu, $I_3 = +1$
   D^0: $c\overline{u}$, $I_3 = -\frac{1}{2}$
   (b) $I = 1$; cud (Σ_c^+), cdd (Σ_c^0)
   (c) (i) strong, (ii) strong, (iii) weak, (iv) weak, (v) forbidden in Standard Model
   (d) No numerical answer