The atomic mass unit is $M_u = 931.494$ MeV/c^2.

The quantum numbers of the photon are $J^{PC} = 1^{--}$. The quantum numbers of the pion are $J^{PC} = 0^{-+}$.

1. (a) Draw a Feynman diagram to represent the weak-interaction process in which an electron is captured by a proton to produce a neutron. (You may treat the proton and neutron as "elementary" particles.) [5 marks]

   (b) Show that an electron-positron pair with orbital angular momentum $L = 0$ and total spin $S = 1$ can annihilate into three photons but not into two. [5 marks]

   (c) Write down an expression that describes the systematic dependence of nuclear radii on atomic number A. Use this to estimate the radius of ^{90}Zr. [4 marks]

   (d) Describe what is meant by the term "magic numbers" in nuclear physics. Explain briefly why these differ from the corresponding numbers in atomic physics. [6 marks]

   (e) Tritium (^{3}H) decays to ^{3}He emitting electrons with a maximum energy of 18.6 keV. The mass excess of ^{3}He is 14931.2 keV/c^2. Find the mass of ^{3}H in atomic mass units. Give your answer to 7 significant figures. (You may neglect nuclear recoil.) [5 marks]

2. (a) Describe what is meant by parity reflection (P) and charge conjugation (C). For each of the known fundamental forces, indicate which of these symmetries and their combination, CP, are respected. [8 marks]

   (b) The $a_1^0(1260)$ meson is observed to decay into a σ meson which has quantum numbers $J^{PC} = 0^{++}$, and a neutral pion. The relative motion of the two final particles is a p wave. The meson has a width $\Gamma \simeq 400$ MeV.

      i. Estimate the lifetime of the $a_1^0(1260)$. State which interaction is most likely to be responsible for the decay of this particle? [3 marks]

      ii. Deduce the J^{PC} quantum numbers of the $a_1^0(1260)$. [7 marks]

      iii. Deduce the relative orbital angular momentum L and total spin S of the quarks in a quark model for the $a_1^0(1260)$. [7 marks]

3. The semi-empirical mass formula can be written in the form:

$$M(A,Z) = Z\,M(^1\mathrm{H}) + (A-Z)M_n - a_v\,A + a_s\,A^{2/3} + a_c\,\frac{Z^2}{A^{1/3}} + a_a\,\frac{(A-2Z)^2}{A} - \delta_p,$$

where

$$\delta_p = \begin{cases} +a_p\, A^{-1/2} & \text{even-even} \\ 0 & \text{odd } A \\ -a_p\, A^{-1/2} & \text{odd-odd} \end{cases} .$$

A good fit to measured masses gives the values $a_c = 0.71$ MeV and $a_a = 23.3$ MeV.

(a) Explain briefly the physical origin of each term, stating any assumptions you have made about the nuclei to which this formula applies. [12 marks]

(b) For a set of nuclei with the same, odd mass number A, show that the most bound nucleus has

$$Z \simeq \frac{A}{2}\left(1 + \frac{a_c A^{2/3}}{4\, a_a}\right)^{-1} .$$

Using the values for the parameters given above, explain why the line of stability follows $N = Z$ for light nuclei but favours a neutron excess for heavier nuclei. [8 marks]

(c) Use your result to predict the most bound isobars with $A = 209$ and $A = 259$. Compare your results with the observations that the most stable element with $A = 209$ is bismuth, $Z = 83$, and the most stable with $A = 259$ is nobelium, $Z = 102$. Comment on any discrepancy you find. [5 marks]

4. (a) Briefly describe the following quantum numbers of quarks and hadrons: baryon number, B, strangeness, S, and isospin third component, I_3. Write down an expression for the electric charge of a hadron in terms of these. Indicate which of these quantities are conserved by each of the fundamental forces. [10 marks]

(b) The Λ^0 baryon is produced in the strong interaction between a π^- meson and a proton. Write down a possible process for this, indicating all the particles in the final state. Draw a quark diagram representing this process. [6 marks]

(c) The Λ^0 decays to a proton and a π^- with a lifetime of about 2.6×10^{-10} s. Which interaction is most likely to be responsible? Give two reasons to support your answer. Draw a quark diagram representing this decay. [6 marks]

(d) The Λ^0 decays to a final $p\pi^-$ state that is observed to be a mixture of s and p waves. Explain briefly what this tells us about the interaction responsible. [3 marks]

NUMERICAL AND BOTTOMLINE ANSWERS

1. (a) No numerical answer
   (b) Use C parity
   (c) 5.4 fm
   (d) No numerical answer
   (e) 3.016049 u

2. (a) No numerical answer
   (b) i. $\sim 2 \times 10^{-24}$ s; strong
       ii. $J^{PC} = 1^{++}$
       iii. $L = S = 1$ (from P, L must be odd; from C, $L + S$ must be even)

3. (a) No numerical answer
   (b) No numerical answer
   (c) $A = 209$: $Z \simeq 82.4$
       $A = 259$: $Z \simeq 98.9$

4. (a) No numerical answer
   (b) $\pi^- + p \rightarrow \Lambda^0 + K^0$ (other more complicated final states are also possible, but they must contain one particle with $S = +1$)
   (c) Weak
   (d) No numerical answer