PHYS 30101 APPLICATIONS OF QUANTUM PHYSICS EXAMPLES 7

Revision: As well as problems similar to the ones on Examples 2 to 6, the exam will contain questions asking you to describe some of the phenomena we have met in this course or to explain the origin of some of the important results. To practise this, you should write brief (one paragraph or so) answers to the following. You should not include any detailed mathematical derivations, but you may quote important formulae.

1. Explain briefly the origin of the formula

$$T = \exp\left[-2\sqrt{\frac{2m}{\hbar^2}\,(V_0 - E)}\,b\right].$$

   for the probability of quantum mechanical tunnelling, defining the quantities involved and stating any approximations you have used or effects you have neglected.

2. Outline how the formula in question 1 can be generalised to tunnelling through a varying barrier $V(x)$, and give the final expression for the tunnelling probability. State any assumptions you have made in obtaining this.

3. Define the term "orthogonality" as applied to functions and describe briefly an example of its role in quantum mechanics.

4. Explain what is meant by a closed shell of a quantum system of trapped particles with degenerate states. Illustrate your answer with examples of "magic numbers" corresponding to closed shells in atoms, nuclei or quantum dots.

5. Explain briefly why the conductance of a quantum dot is nonzero only for certain values of the gate voltage, when the drain and source are held at the same voltage. How does this picture change if the drain and source are held at different voltages?

6. Describe what is meant by "Coulomb blockade" in a quantum dot and give the analogous effect in atomic physics.

7. Explain briefly the origin of the sharp spikes in the conductance of a quantum wire.

8. A system is decribed by the Hamiltonian

$$\widehat{H} = \widehat{H}_0 + \lambda \widehat{H}_1,$$

   where the eigenvalues and eigenfunctions of $\widehat{H}_0$ are known and λ is a small parameter. Write down an expression for the eigenvalues of $\widehat{H}$ to first order in λ. Explain briefly why your expression makes physical sense. State any assumptions you have made about the eigenfunctions of $\widehat{H}_0$.

9. Define the magnetic g-factor of the electron and give its value. What is special about the closest integer value for g?

10. Write down the pattern of eigenvalues of angular momentum operator $\widehat{J}_z$ for some system that is in an eigenstate of $\widehat{\mathbf{J}}^2$, with total quantum number J. Give also the eigenvalue of $\widehat{\mathbf{J}}^2$. Explain briefly why this pattern is consistent for half-integer as well as integer values of J. Explain also why only integer values are possible if J is the orbital angular momentum of some rotating system.

11. Two angular momenta with quantum numbers L and S are combined to form a total angular momentum with quantum number J. Explain briefly why the values of J run from $|L - S|$ to $L + S$ in steps of one.

12. Explain briefly the origin of spin-orbit coupling. You should give, with justification, the sign of the interaction for a one-electron atom.

13. Define the Landé g-factor and, without giving a detailed algebraic derivation, give an explanation for its origin. State any assumptions you have made.

14. Explain what is meant by "entanglement" in quantum mechanics.

15. Describe briefly a practical application of entanglement.