PHYS 30101 APPLICATIONS OF QUANTUM PHYSICS EXAMPLES 1

Questions 1 to 6 are short "warm up" questions to get you thinking about the ideas of quantum mechanics again, and where they are important. Question 7 is a longer problem, revising methods you used last year.

1. What are the wavelengths (in nm) of (a) a photon with an energy of 2 eV, (b) an electron with an energy of 2 eV? What things have you seen in physics with these sizes?

2. The quantum-mechanical momentum operator in one dimension is defined by

$$\widehat{p} = -\mathrm{i}\,\hbar\,\frac{\partial}{\partial x}.$$

   (a) Show that the wave function

$$\psi_a(x) = A\mathrm{e}^{\mathrm{i}kx}$$

   is an eigenfunction of momentum. What is its eigenvalue? If you were to measure the momentum of a particle described by this wave function, what value would you get?

   (b) Show that the wavefunction

$$\psi_b(x) = B\cos(kx)$$

   is not a momentum eigenfunction. If you were to measure the momentum of a particle described by this wave function, what possible values could you get? What can you say the probabilities of getting each of these values?

3. Two waves have the forms

$$\begin{array}{lrcl} \text{(a)} & \psi(\mathbf{r},t) & = & A\exp[\mathrm{i}(kx-\omega t)], \\ \text{(b)} & \psi(\mathbf{r},t) & = & A\exp[-\mathrm{i}(ky+\omega t)], \end{array}$$

   where ω and k are both positive. What is the direction of travel of each of these waves? Justify your answers.

4. The operator for the z-component of the angular momentum is

$$\widehat{L}_z = -\mathrm{i}\,\hbar\,\frac{\partial}{\partial \phi},$$

   where ϕ is the angle around the z-axis. Write down its eigenfunctions and eigenvalues. Explain why these eigenvalues are quantised.

5. Last year you met three types of system with quantised energy levels. In each case the energy of a state can be written in terms of a whole number n. The energies of the three quantum systems have the forms:

$$\begin{array}{lrcl} \text{(a)} & E_n & \propto & n^2, \\ \text{(b)} & E_n & \propto & n+\tfrac{1}{2}, \\ \text{(c)} & E_n & \propto & -\dfrac{1}{n^2}. \end{array}$$

What are these systems? What are the allowed values of n in each case? Supply the missing constant of proportionality for each expression.

6. The state of a particle is described by a solution of the TISE, $\psi_n(x)$, which has energy eigenvalue E_n. Write down the corresponding (separable) solution $\Psi(x,t)$ of the TDSE and find the corresponding probability density $P(x,t) = |\Psi(x,t)|^2$. Use your result to explain why this solution is known as a "stationary state".

7. A particle of mass M is trapped in an infinite square well of width a,

$$V(x) = \begin{cases} 0 & \text{for } \; 0 < x < a \\ \infty & \text{otherwise} \end{cases}.$$

(a) Show that its energy eigenfunctions are given by

$$\psi_n(x) = \begin{cases} B\sin\left(\dfrac{n\pi x}{a}\right) & \text{for } \; 0 < x < a \\ 0 & \text{otherwise} \end{cases},$$

where B is a constant, and the corresponding energy eigenvalues are

$$E_n = \frac{n^2\pi^2\hbar^2}{2Ma^2},$$

where $n = 1, 2, 3, \ldots$.

(b) Sketch the first three of these eigenfunctions.

(c) Show that these functions are normalised if we set

$$B = \sqrt{\frac{2}{a}}.$$

(d) Show that, for any of these eigenfunctions, the probability of finding the particle in the region $0 < x < a/2$ is $1/2$.

(e) Show that the expectation value of x for any of these eigenfunctions is

$$\langle x \rangle = \frac{a}{2}.$$

Give a physical explanation of this result.

(f) Show that two of these eigenfunctions $\psi_n(x)$ and $\psi_m(x)$ with $n \neq m$ are orthogonal, that is

$$\int_0^a \psi_n^*(x)\psi_m(x)\,\mathrm{d}x = 0.$$