

## Lecture 9

### Perturbation theory

#### Full Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{H}_1$$

- $\hat{H}_0$  simple, known eigenstates:  $\hat{H}_0|\phi_n\rangle = E_n^{(0)}|\phi_n\rangle$
- $\hat{H}_1$ : small “perturbation”

To first order in  $\hat{H}_1$

- eigenvalues of  $\hat{H}$  shifted by expectation value of  $\hat{H}_1$

$$E_n \simeq E_n^{(0)} + \langle \hat{H}_1 \rangle$$

where

$$\langle \hat{H}_1 \rangle = \langle \phi_n | \hat{H}_1 | \phi_n \rangle \left( = \int \phi_n^*(\mathbf{r}) \hat{H}_1 \phi_n(\mathbf{r}) d^3\mathbf{r} \right)$$

- but be careful if  $\hat{H}_0$  has degenerate eigenvalues
- need to find states not mixed by  $\hat{H}_1$  first