Lecture 8

Orthogonality

Eigenfunctions of Hermitian operator $\widehat{A}$

$$\widehat{A}\phi_n(\mathbf{r}) = a_n \phi_n(\mathbf{r})$$

- eigenvalues a_n are real (possible measurements of A)
- eigenfunctions $\phi_n(\mathbf{r})$ are orthogonal

$$\int \phi_m^*(\mathbf{r})\,\phi_n(\mathbf{r})\,\mathrm{d}^3\mathbf{r} = 0 \qquad \text{unless} \quad a_m = a_n$$

General state $\Psi(\mathbf{r}, t)$: superposition of eigenfunctions
($\sim$ Fourier series) with coefficients

$$c_n(t) = \int \phi_n^*(\mathbf{r})\,\Psi(\mathbf{r}, t)\,\mathrm{d}^3\mathbf{r}$$

Probability of getting result a_n at time t

$$P_n(t) = |c_n(t)|^2$$

Dirac notation

We denote the quantum state of a system by the "ket"

$$|\psi\rangle$$

instead of writing down a wave function that depends on all the positions of its particles $\psi(\mathbf{r}_1, \mathbf{r}_2, \ldots)$

(gets tedious for more than 2 particles and isn't possible for particles with intrinsic degrees of freedom like spin)

Overlap of two states is written $\langle\chi|\psi\rangle$ (sort of like $\mathbf{A}\cdot\mathbf{B}$ for vectors)

For single particle, this unpacks to

$$\langle\chi|\psi\rangle = \int \chi^*(\mathbf{r})\psi(\mathbf{r})\,\mathrm{d}^3\mathbf{r}$$

Orthogonality: $\langle\phi_n|\phi_m\rangle = 0$ unless $a_n = a_m$

Normalisation: $\langle\phi_n|\phi_n\rangle = 1$