Lecture 20A

Can we explain quantum mechanics with hidden classical variables?

Two electrons in state

$$\psi_{00}(1,2) = \frac{1}{\sqrt{2}}\Big(\alpha_z(1)\,\beta_z(2) - \beta_z(1)\,\alpha_z(2)\Big)$$

- always have opposite spins along any axis
- measure spins along axes at 120°

$$\widehat{S}_a = \widehat{S}_z \qquad \widehat{S}_b = \widehat{\mathbf{S}} \cdot \Big(\sin(2\pi/3)\,\mathbf{i} + \cos(2\pi/3)\,\mathbf{k}\Big)$$

- get (for example) $S_a = +\hbar/2$ for particle 1 and $S_b = +\hbar/2$ for particle 2

Classical (local) hidden variables $\longrightarrow$ probability between 0% and 67%
(symmetry argument based on three analysers all at 120°)

Quantum mechanics: entangled state

$$\psi_{00}(1,2) = \frac{1}{\sqrt{2}}\Big(\alpha_z(1)\,\beta_z(2) - \beta_z(1)\,\alpha_z(2)\Big)$$

$\longrightarrow$ if we get $S_z = +\hbar/2$ for 1, know 2 is in state

$$\beta_z = c_1\,\alpha_b + c_2\,\beta_b$$

where eigenstates of $\widehat{S}_b$ are

$$\alpha_b = \frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \qquad \beta_b = \begin{pmatrix} \sqrt{3} \\ -1 \end{pmatrix}$$

- probability of getting $S_b = +\hbar/2$ for 2 is

$$P = |c_1|^2 = |\alpha_b{}^\dagger\,\beta_z|^2 = 75\%$$

$\longrightarrow$ incompatible with hidden variables

Experiments: quantum mechanics is right!