Lecture 2

Time-independent Schrödinger equation in 1D (constant potential)

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V_0\psi = E\psi$$

- energy $E > V_0 \longrightarrow$ waves

$$\psi(x) = A\mathrm{e}^{\mathrm{i}kx} + B\mathrm{e}^{-\mathrm{i}kx}, \qquad k^2 = \frac{2m}{\hbar^2}(E - V_0)$$

- $E < V_0 \longrightarrow$ tunnelling

$$\psi(x) = C\mathrm{e}^{\beta x} + D\mathrm{e}^{-\beta x}, \qquad \beta^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

Simple barrier: $V = V_0$ for $0 < x < b$

- $x = 0,\ b$: edges of classical forbidden zone for $E < V_0$
- "tunnelling factor" for wide barrier, $\beta b >> 1$

$$T = \exp\left[-2b\sqrt{\frac{2m}{\hbar^2}(V_0 - E)}\right]$$

- dominant factor, strong (exponential) dependence on E, b
  (full probability contains factors accounting for reflection at edges
  – less important, weaker dependence on E)

Useful trick for numerical work

- work in sensible units, such as eV and nm for atomic physics or MeV and fm for nuclear
- write your answer in terms terms of rest energies, mc^2
- use $\hbar c \simeq 200$ eV nm $= 200$ MeV fm $= 200$ TeV zm and $\alpha = e^2/(4\pi\varepsilon_0\hbar c) = 1/137$