Lecture 18

Zeeman effect: atom in external magnetic field $\mathbf{B} = B\mathbf{e}_z$

$$\widehat{H}_{\mathrm{mag}} = \frac{eB}{2m}\left(\widehat{L}_z + 2\widehat{S}_z\right)$$

Weak field: spin-orbit energy much larger than magnetic
$\longrightarrow$ work with states of definite $|\mathbf{L}|^2$, $|\mathbf{S}|^2$, $|\mathbf{J}|^2$ and J_z
Using first-order perturbation theory

$$E_{\mathrm{mag}} = \frac{eB}{2m}\left(M_J\hbar + \langle \widehat{S}_z \rangle\right)$$

Uncertainty principle $\longrightarrow$ only component of $\mathbf{S}$ along $\mathbf{J}$ contributes

$$\langle \widehat{S}_z \rangle = \frac{\langle (\widehat{\mathbf{J}} \cdot \widehat{\mathbf{S}})\widehat{J}_z \rangle}{|\mathbf{J}|^2} = \frac{M_J}{J(J+1)\hbar} \langle \widehat{\mathbf{J}} \cdot \widehat{\mathbf{S}} \rangle$$

[in terms of squares $\widehat{\mathbf{J}} \cdot \widehat{\mathbf{S}} = \left(\widehat{\mathbf{J}}^2 + \widehat{\mathbf{S}}^2 - \widehat{\mathbf{L}}^2\right)/2$]

Defining the Landé g-factor

$$g_J = 1 + \frac{\langle \widehat{\mathbf{J}} \cdot \widehat{\mathbf{S}} \rangle}{J(J+1)} = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

magnetic energies are

$$E_{\mathrm{mag}} = g_J \frac{e\hbar}{2m} B M_J$$

$\longrightarrow$ ladder of equally spaced states

Strong field: magnetic energy much larger than spin-orbit
$\longrightarrow$ go back to $\widehat{H}_{\mathrm{mag}}$ and work with states of definite L_z and S_z