Lecture 17

Electron orbiting nucleus feels magnetic field $\mathbf{B} \propto \mathbf{L}$

$\longrightarrow$ spin-orbit coupling

$$\widehat{H}_{\mathrm{so}} = f(r)\,\widehat{\mathbf{L}}\cdot\widehat{\mathbf{S}}$$

In terms of squares of angular momentum operators

$$\widehat{\mathbf{L}}\cdot\widehat{\mathbf{S}} = \frac{1}{2}\left[\widehat{\mathbf{J}}^2 - \widehat{\mathbf{L}}^2 - \widehat{\mathbf{S}}^2\right] \qquad \text{where} \quad \widehat{\mathbf{J}} = \widehat{\mathbf{L}} + \widehat{\mathbf{S}}$$

Eigenstates of $\widehat{\mathbf{J}}^2$, $\widehat{J}_z$ are eigenstates of $\widehat{\mathbf{L}}\cdot\widehat{\mathbf{S}}$

$$\widehat{\mathbf{L}}\cdot\widehat{\mathbf{S}}\,\psi_{JM_J} = \frac{\hbar^2}{2}\Big[J(J+1) - L(L+1) - S(S+1)\Big]\psi_{JM_J}$$

$\longrightarrow$ not mixed by $\widehat{H}_{\mathrm{so}}$ (unlike degenerate eigenstates of $\widehat{L}_z$, $\widehat{S}_z$)

Using first-order perturbation theory

$$E_J = \mathcal{E}\,\frac{1}{2}\Big[J(J+1) - L(L+1) - S(S+1)\Big]$$

where $\mathcal{E} = \langle f(r)\rangle\,\hbar^2 > 0$ and so smaller J has lower energy