Lecture 16

Manipulating spins

TDSE for electron spin in a constant magnetic field $\mathbf{B}_0 = (0, 0, B_0)$ and a rotating field $\mathbf{B}_1 = (B_1 \cos\omega t, B_1 \sin\omega t, 0)$

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \frac{eg\hbar}{4m}\begin{pmatrix} B_0 & B_1\mathrm{e}^{-\mathrm{i}\omega t} \\ B_1\mathrm{e}^{\mathrm{i}\omega t} & -B_0 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

- does not separate

Use fact that for $B_1 = 0$ spin precesses around z axis at rate $2\omega_0$ where $\omega_0 = egB_0/4m$

Write $c_1(t) = a_1(t)\,\mathrm{e}^{-\mathrm{i}\omega_0 t}$, $c_2(t) = a_2(t)\,\mathrm{e}^{\mathrm{i}\omega_0 t}$
where $a_{1,2}(t)$ describe additional time dependence produced by B_1

TDSE becomes

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \frac{eg\hbar}{4m}\begin{pmatrix} 0 & B_1\mathrm{e}^{\mathrm{i}(2\omega_0-\omega)t} \\ B_1\mathrm{e}^{-\mathrm{i}(2\omega_0-\omega)t} & 0 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Resonance condition: $\omega = 2\omega_0$

- field B_1 rotates at same rate as spin
- looks like constant field along rotating x' axis

$\longrightarrow$ spin precesses around x' axis at rate $2\omega_1$ where $\omega_1 = egB_1/4m$

(electron oscillates between spin-up and spin-down)