Lecture 15

Qbit – unit of quantum information (two-state system)

Examples: electron spin states

$$\alpha_z \quad (S_z = +\hbar/2) \qquad \text{for “1”}$$
$$\beta_z \quad (S_z = -\hbar/2) \qquad \text{for “0”}$$

Linear superposition $\alpha_x = (\alpha_z + \beta_z)/\sqrt{2}$
$\rightarrow$ 50% chances of 0 or 1 but 100% chance of $S_x = +\hbar/2$

Photon polarisation states: vertical and horizontal

$$V = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad H = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

or diagonal $(+45^\circ)$ and antidiagonal (-45°)

$$D = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

or right- and left-handed circular: $\pm\pi/2$ phase difference $(\pm \mathrm{i})$

Double quantum dot: electron in lowest state of left or right dot

$$L = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad R = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Energy difference between dots: ε, tunnelling coupling: $-\Delta/2$
$\longrightarrow 2 \times 2$ Hamiltonian

$$\widehat{H} = \frac{1}{2}\begin{pmatrix} \varepsilon & -\Delta \\ -\Delta & -\varepsilon \end{pmatrix} = \frac{1}{2}\left(\varepsilon\sigma_3 - \Delta\sigma_1\right)$$

(just like spin in magnetic field with x and z components)