Lecture 14

Addition of angular momenta: for example $\widehat{\mathbf{J}} = \widehat{\mathbf{L}} + \widehat{\mathbf{S}}$

Eigenvalues of $\widehat{\mathbf{J}}^2$ are $|\mathbf{J}|^2 = J(J+1)\,\hbar^2$ where

$$J = L+S,\ L+S-1,\ \ldots,\ |L-S|$$

Eigenvalues of $\widehat{J}_z$ are $J_z = M_J\,\hbar$ where $M_J = +J,\ J-1,\ \ldots,\ -J$

States with definite J, M_J are linear superpositions of ones with definite M_L, $M_S \rightarrow$ total numbers of states must match:

$$\sum_{J=|L-S|}^{L+S} (2J+1) = (2L+1)(2S+1)$$

[To construct states: start from one with $M_L = L$, $M_S = S$; hit it repeatedly with $\widehat{J}_-$ to build ladder with $J = L+S$; use orthogonality to get second state with $M_J = L+S-1$ and start a second ladder with it; repeat until independent states (and you) are exhausted]