Lecture 13

Charged particle in magnetic field $\longrightarrow$ Hamiltonian

$$\widehat{H} = -\widehat{\mu} \cdot \mathbf{B}$$

Magnetic moments of electron, charge $q = -e$

- orbital: $\widehat{\mu} = -\dfrac{e}{2m}\widehat{\mathbf{L}}$
- intrinsic: $\widehat{\mu} = -\dfrac{eg}{2m}\widehat{\mathbf{S}}$
- magnetic g-factor of electron $g \simeq 2$

Electron in field $\mathbf{B} = B\mathbf{e}_z$: energy eigenvalues

- orbital: $E = \dfrac{e\hbar}{2m} B m_l$ where $m_l = -l, \ldots, +l$

$\longrightarrow$ ladder of equally spaced states

- intrinsic: $E = \pm \dfrac{eg\hbar}{4m} B$

$\longrightarrow$ splitting twice that of orbital states ($g \simeq 2$)

TDSE for electron spin in field $\mathbf{B} = B\mathbf{e}_z$

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \frac{eg}{2m}\widehat{\mathbf{S}}\cdot\mathbf{B}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \frac{eg\hbar}{4m}B\sigma_3\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

Solution

$$\begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} = \begin{pmatrix} c_1(0)\,\mathrm{e}^{-\mathrm{i}\omega t} \\ c_2(0)\,\mathrm{e}^{+\mathrm{i}\omega t} \end{pmatrix}$$

$\longrightarrow \langle\widehat{\mathbf{S}}\rangle$ precesses around $\mathbf{B}$ at rate $\omega_p \simeq eB/m = \omega_c$ (cyclotron freq)

$g = 2$ is special: spin and orbital motion precess together