Lecture 12

Intrinsic spin of electron (proton, neutron, muon, ...)

Angular-momentum quantum number $s = \frac{1}{2} \rightarrow$ eigenvalues

$$|\mathbf{S}|^2 = \frac{1}{2}\left(\frac{1}{2}+1\right)\hbar^2 \qquad S_z = m_s\hbar = \pm\frac{1}{2}\hbar$$

Basis states represented by two-component "spinors"

$$\alpha_z = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \beta_z = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Spin angular-momentum operators

$$\widehat{S}_x = \frac{\hbar}{2}\sigma_1 \qquad \widehat{S}_y = \frac{\hbar}{2}\sigma_2 \qquad \widehat{S}_z = \frac{\hbar}{2}\sigma_3$$

in terms of 2×2 Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Eigenvectors of $\widehat{S}_z$: α_z, β_z, with eigenvalues: $+\hbar/2$, $-\hbar/2$

General spinor

$$\gamma = c_1\,\alpha_z + c_2\,\beta_z = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$|c_1|^2$, $|c_2|^2$: probabilities of getting $S_z = +\hbar/2, -\hbar/2$

Expectation value of operator (matrix) $\widehat{A}$ in state γ

$$\langle \widehat{A} \rangle = \gamma^\dagger \widehat{A} \gamma$$

Hermitian conjugation (complex conjugation and transpose)

$$\gamma^\dagger = \begin{pmatrix} c_1^* & c_2^* \end{pmatrix}$$