Lecture 10

Quantum wire (particle trapped in 2D, free in third)

Energy eigenvalues

$$E = \frac{\hbar^2 k_z^2}{2m^*} + E_m$$

E_m: energy eigenvalues of (trapped) transverse motion

Density of states in energy, for electrons in transverse state m

$$\frac{\mathrm{d}n}{\mathrm{d}E} \text{ [or } g(E)] = \frac{L}{\pi}\left(\frac{2m^*}{\hbar^2}\right)^{1/2} \frac{1}{\sqrt{E - E_m}} \qquad \text{for } E > E_m$$

Spikes at each $E_m \longrightarrow$ peaks in electrical conductance
(unlike bulk conductor: smooth density of states $\propto \sqrt{E}$)

Cylindrical wire: periodic wave functions $\longrightarrow$ m integer

$$E_m = \frac{\hbar^2 m^2}{2m^* R^2}$$

Graphene

Single-atom-thick sheet of carbon in hexagonal pattern

Effective mass of electron $m^* = 0$

$\longrightarrow$ relativistic form for energies

$$E = v_F\,\hbar\,|\mathbf{k}| \qquad \text{(like } E = c\,|\mathbf{p}|\text{)}$$

but with Fermi velocity

$$v_F \simeq 10^6\ \mathrm{ms}^{-1} = \frac{1}{300}\,c \qquad \text{instead of } c$$

Carbon nanotube

Cylinder of graphene, radius $\sim$ 1–2 nm

Energy eigenvalues of transverse motion

$$E_m = v_F\,\hbar\,\frac{|m|}{R} \qquad \text{with } m \text{ integer (periodic waves)}$$

Full energies: $E = v_F\,\hbar\,|\mathbf{k}| = \sqrt{v_F^2\,\hbar^2\,k_z^2 + E_m^2}$

Density of states has spikes at equally spaced energies E_m