LECTURE 21

Normal modes on surface of sphere

- orthogonal eigenfunctions, e.g.

$$\int_0^{\pi} P_l(\cos\theta)\, P_k(\cos\theta) \sin\theta\, d\theta = 0 \qquad l \neq k$$

from $dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$

→ build waves as Legendre series etc.

Waves inside sphere

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \phi^2} = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$

- periodic in ϕ
- regular at $r = 0$, $\theta = 0, \pi$

Normal modes

$$f(r, \theta, \phi, t) = j_l(kr)\, Y_{lm}(\theta, \phi)\left[A \cos\omega t + B \sin\omega t\right]$$

j_l: spherical Bessel function

$\omega = ck$