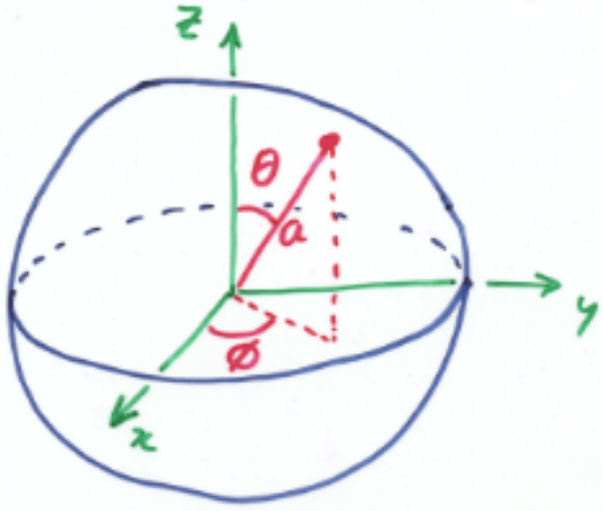
LECTURE 20

Waves on a sphere



$$\nabla^2 f = \frac{1}{a^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial f}{\partial\theta}\right) + \frac{1}{a^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2} = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$

Boundary conditions:

- periodic in ϕ

$$f(\theta,\phi,t) = f(\theta,\phi+2\pi,t)$$

- regular at $\theta = 0, \pi$

Normal modes (spherical harmonics)

$$f(\theta,\phi,t) = P_l^m(\cos\theta)\begin{Bmatrix}\cos m\phi \\ \sin m\phi\end{Bmatrix}(A\cos\omega_l t + B\sin\omega_l t)$$

↑ associated Legendre fn.

l, m whole numbers, $|m| \le l$

$$\omega_l = \frac{c}{a}\sqrt{l(l+1)}$$