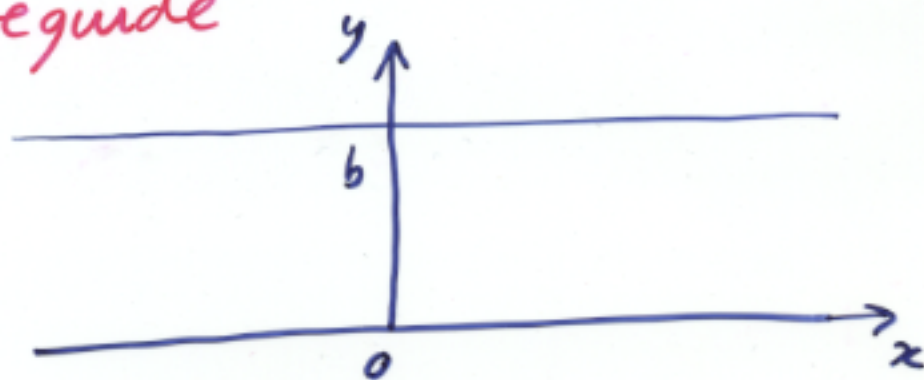


LECTURE 18

Waveguide



Wave equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2}$$

subject to $\phi(x, 0, t) = \phi(x, b, t) = 0$

Travelling-wave solutions

$$\phi(x, y, t) = A \sin \frac{n\pi y}{b} e^{i(k_x x - \omega t)}$$

$n = 1, 2, 3, \dots$

(k_x : anything)

Dispersion relation

$$\omega = c \sqrt{k_x^2 + \frac{n^2 \pi^2}{b^2}}$$

Group velocity (in x direction)

$$v_g = \frac{d\omega}{dk_x} = c \frac{k_x}{\sqrt{k_x^2 + n^2 \pi^2 / b^2}}$$

Cut-off freq. for n -th mode: $\omega_{nc} = c \frac{n\pi}{b}$

$\omega < \omega_{nc}$: evanescent waves ($e^{-\alpha x}$)