LECTURE 17

Rectangular membrane, fixed edges

Wave equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2}$$

subject to $\phi = 0$ on edges

Normal modes (separable solutions → eigenfunctions)

$$X_n(x) Y_m(y) = \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b}$$

$$n = 1, 2, 3, \ldots, \quad m = 1, 2, 3, \ldots$$

Frequencies

$$\omega_{nm} = c \sqrt{\frac{n^2\pi^2}{a^2} + \frac{m^2\pi^2}{b^2}}$$

Nodal lines: zero amplitude for all t

Degenerate modes: same frequency