LECTURE 16

Bessel's equation $\quad 0 \le x < \infty$

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - m^2)\, y = 0$$

Power series solution

$$y = \sum_{n=0}^{\infty} a_n x^{n+s}$$

with $s = \pm m$ and

$$a_{n+2} = -\frac{1}{(n+s)^2 - m^2}\, a_n$$

Large n:

$$a_{n+2} \simeq -\frac{1}{n^2}\, a_n$$

- like series for cosine, sine
→ oscillatory functions

$s = +m$: regular at $x = 0$

$s = -m$: singular at $x = 0$

Solutions: Bessel functions $J_m(x)$, $N_m(x)$

↑ regular ↑ irregular