LECTURE 15

Legendre polynomials: eigenfunctions of

$$\frac{d}{dx}\left[(1-x^2)\frac{dP_l}{dx}\right] + l(l+1)P_l = 0$$

with b.c. that $P_l(x)$ is finite at $x = \pm 1$

Eigenvalue: $l(l+1)$ $\quad l = 0, 1, 2 \ldots$

(orbital ang. mom. $\underline{\hat{L}}^2 \psi = l(l+1)\hbar^2 \psi$)

Orthogonal on $-1 \leq x \leq +1$

$$\int_{-1}^{1} P_l(x) P_m(x)\,dx = 0 \qquad m \neq l$$

$$\left(\frac{2}{2l+1} \qquad m = l\right)$$

Use to build Legendre series

$$f(x) = \sum_{l=0}^{\infty} c_l P_l(x)$$

$$c_l = \frac{2l+1}{2}\int_{-1}^{1} f(x) P_l(x)\,dx$$