LECTURE 14

Legendre's equation

$-1 \le x \le +1$

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + l(l+1)y = 0$$

Power series solution

$$y = a_0 + a_1 x + a_2 x^2 + \dots$$

with

$$a_{n+2} = -\frac{l(l+1) - n(n+1)}{(n+2)(n+1)} a_n$$

For $y(x)$ finite at $x = \pm 1$ need

$l = 0, 1, 2 \dots$

so even or odd series terminates at $n = l$

Solutions: Legendre polynomials $P_l(x)$

[E.g. $P_0(x) = 1$, $P_1(x) = x$,

$P_2(x) = \frac{1}{2}(3x^2 - 1)$]