LECTURE 13

Taylor series

$$f(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n$$

Coefficients

$$a_n = \frac{1}{n!} f^{(n)}(x_0)$$

Converges if ratios of terms always < 1

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} (x - x_0) \right| < 1$$

(need to look at $n+2 : n$ if series is even or odd)

Terms are linearly independent

$$\sum_{n=0}^{\infty} a_n (x - x_0)^n = 0$$

if and only if $a_n = 0$ for all n

[Approximations:

$$f(x) \simeq f(x_0) + f'(x_0)(x - x_0)$$

or at stationary point, $f'(x_0) = 0$

$$f(x) \simeq f(x_0) + \tfrac{1}{2} f''(x_0)(x - x_0)^2$$
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