LECTURE 9

The heat-flow equation

$$\nabla^2 f = \frac{1}{D}\frac{\partial f}{\partial t}$$

diffusivity $D = \frac{K}{c\rho}$

Insulated boundary

$$\underline{n}\cdot\nabla f\big|_S = 0$$

$\underline{n}$: normal to S

Bar with insulated ends

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{D}\frac{\partial f}{\partial t}$$

subject to

$$\left.\frac{\partial f}{\partial x}\right|_0 = \left.\frac{\partial f}{\partial x}\right|_L = 0$$

General solution

$$f(x,t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\frac{n\pi x}{L}\, e^{-\gamma_n t}$$

relaxation rate $\gamma_n = D\left(\frac{n\pi}{L}\right)^2$

Long times

$$f(x,t) \simeq A_0 + A_1 \cos\frac{\pi x}{L}\, e^{-\gamma_1 t} \xrightarrow[t\to\infty]{} A_0$$

average temp.