LECTURE 8

Laplace's equation on a square

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

B.C.'s $\phi(x,0) = \phi(x,L) = 0$

Separate variables $\phi(x,y) = X(x)\,Y(y)$

$$\frac{d^2X}{dx^2} = k^2 X$$

$$\frac{d^2Y}{dy^2} = -k^2 Y$$

$$Y(0) = Y(L) = 0$$

eigenvalue problem

$$X_n(x) = A_n e^{\frac{n\pi x}{L}} + B_n e^{-\frac{n\pi x}{L}}$$

$$Y_n(y) = \sin\frac{n\pi y}{L}$$

$$k_n^2 = \frac{n^2\pi^2}{L^2}$$

General solution: sum of separable

$$\phi(x,y) = \sum_{n=1}^{\infty} \left(A_n e^{\frac{n\pi x}{L}} + B_n e^{-\frac{n\pi x}{L}} \right) \sin\frac{n\pi y}{L}$$

↑ ↑

Use Fourier series to find these from b.c.'s at $x = 0, L$