

LECTURE 5

Fourier series for general function
on $-L \leq x \leq L$

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$$

with

$$a_0 = \frac{1}{2L} \int_{-L}^L f(x) dx \quad \text{note extra } \frac{1}{2}$$

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx \quad n = 1, 2, 3, \dots$$

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx \quad n = 1, 2, 3, \dots$$

Relies on orthogonality of sines & cosines

$$\text{e.g. } \int_{-L}^L \cos \frac{m\pi x}{L} \sin \frac{n\pi x}{L} dx = 0$$