LECTURE 3

Wave eqn. for string separates into

$$\frac{d^2X}{dx^2} = -k^2X \qquad \frac{d^2T}{dt^2} = -k^2c^2T$$

$$X(0) = X(L) = 0$$

Eigenvalue problem

→ eigenfunctions (normal modes)

$$X_n(x) = \sin\frac{n\pi x}{L} \qquad n = 1, 2, 3 \ldots$$

with eigenvalues (k: wavenumber)

$$k_n^2 = \frac{n^2\pi^2}{L^2}$$

$$T_n(t) = A_n \cos\frac{n\pi ct}{L} + B_n \sin\frac{n\pi ct}{L}$$

General solution of wave equation

- linear superposition of separable solutions

$$\phi(x,t) = \sum_n X_n(x)\, T_n(t)$$

$$= \sum_{n=1}^{\infty} \sin\frac{n\pi x}{L}\left(A_n \cos\frac{n\pi ct}{L} + B_n \sin\frac{n\pi ct}{L}\right)$$

↑ from initial conditions ↑