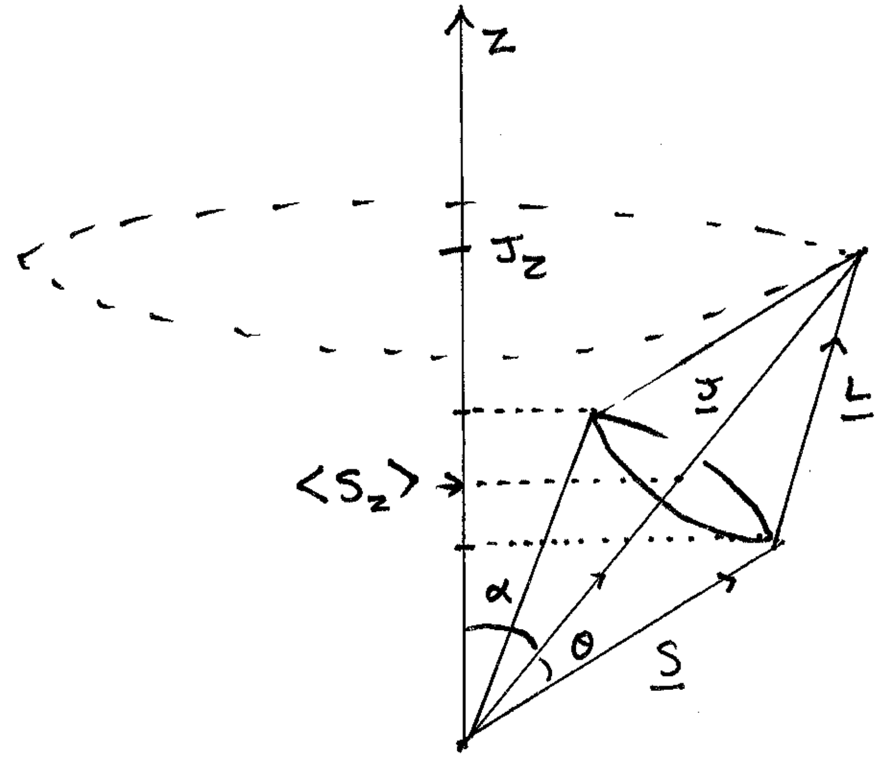
Semi-classical derivation of $\langle S_z \rangle$ for weak-field Zeeman effect



$$\langle S_z \rangle = \frac{1}{2}\left(\cos(\theta+\alpha) + \cos(\theta-\alpha)\right)|S|$$

$$= \cos\theta \cos\alpha \, |S|$$

$$= \frac{S \cdot J}{|J|}\,\frac{J_z}{|J|} \qquad = J_z \frac{S \cdot J}{|J|^2}$$

PTO

Some matrix elements for the Stark effect

$\langle 100 | z^2 | 100 \rangle$

$$= \int | R_{100}(r) \, Y_0^0(\theta,\phi) |^2 (r^2 \cos^2\theta) \, r^2 \, d\Omega \, dr$$

$$= \frac{4}{a_0^3} \int_0^\infty e^{-2r/a_0} r^4 \, dr \quad \times \quad \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^\pi \cos^2\theta \sin\theta \, d\theta$$

* $$= \frac{4}{a_0^3} \times 4! \left(\frac{a_0}{2}\right)^5 \quad \times \quad \frac{1}{3} \qquad \Big| \quad \left[-\frac{1}{3}\cos^3\theta\right]_0^\pi$$

$$= a_0^2$$

$$\langle 210 | z | 200 \rangle = \int R_{210} \, R_{200} \, Y_1^0 \, Y_0^0 \left(\sqrt{\frac{4\pi}{3}} \, Y_1^0 \, r\right) dV$$

$$= \frac{2}{\sqrt{3}} \cdot \frac{1}{(2a_0)^3} \int \frac{r}{a_0}\left(1 - \frac{r}{2a_0}\right) e^{-r/a_0} \, r^3 \, dr \quad \times \frac{1}{\sqrt{3}}$$

* $$= \frac{1}{12 a_0^3} \left(\frac{4!}{a_0} a_0^5 - \frac{1}{2} \times \frac{5!}{a_0^2} a_0^6 \right)$$

$$= -3a_0$$

* use $\displaystyle\int_0^\infty r^n e^{-\alpha r} \, dr = (-1)^n \frac{d^n}{d\alpha^n} \left(\int_0^\infty e^{-\alpha r} \, dr \right) = \frac{n!}{\alpha^{n+1}}$