Guide to the exam 2018-19

This is the fifth year of the course "Mathematical Fundamentals of Quantum Mechanics". This is the second year of a revised syllabus; see below.

The exam will be of the standard format, with a compulsory question 1 consisting of several short sections, and a choice of 2 from 3 long questions. There will be a formula sheet, the current version of which is reproduced at the end of this document and which has not changed from last year. Tables of Clebsch-Gordan coefficients will be provided if required, and will use the format of the PDG tables. Past exams also give a good guide as to the kind of formulae that you are expected to know (for example Ehrenfest's theorem and the 1st order energy shift in perturbation theory, not to mention material from previous relevant courses such as the solutions of the infinite square well).

A well as past papers, the examples sheets (excluding the first, revision, sheet) for the current course provide the best guide to what I consider important.

Compared to the year before last, the following changes have been made:

- Vector spaces are no longer taught in this course
- WKB and the variational approach (section 3) have been added

Vector spaces for its own sake was never a major exam topic, but some parts of question 1 on old papers would not be asked this year. In particular there was was often a question about identifying Hermitian and unitary matrices or operators, which I would not ask now. (1b) of the 2015 exam is likewise no longer specifically part of the syllabus and (1d) of that exam and (1e) of the 2014 exam are more marginal than they were.

WKB and the variational approach was taught in PHYS30201 "Advanced Quantum Mechanics I" (last run 2012/13) for which I was the lecturer. From the exams of that course, I list suitable questions on all areas of the current syllabus below.

2013 All except (1d,e)

2012 (1a), (1c-e), (2a), (3), (4) in part (no "allowed transitions").

2011 (1a-d), (3)

2010 (1a-c), (2) first half only (no time-dependent perturbations), (3), (4).

You may assume the following formulae in any question if proof is not explicitly requested:

Harmonic Oscillator:

$$\hat{a} = \frac{1}{\sqrt{2}}\left(\frac{\hat{x}}{x_0} + i\frac{x_0}{\hbar}\hat{p}\right), \quad \hat{a}^\dagger = \frac{1}{\sqrt{2}}\left(\frac{\hat{x}}{x_0} - i\frac{x_0}{\hbar}\hat{p}\right) \quad \text{and } [\hat{a}, \hat{a}^\dagger] = 1, \qquad \text{where } x_0 = \sqrt{\frac{\hbar}{m\omega}}.$$

$$\hat{a}|n\rangle = \sqrt{n}|n{-}1\rangle \qquad \text{and} \qquad \hat{a}^\dagger|n\rangle = \sqrt{n{+}1}|n{+}1\rangle.$$

Wave functions:

$$\phi_n(x) = (2^n n!)^{-1/2} H_n(\tfrac{x}{x_0}) \times (\pi x_0^2)^{-1/4} \exp\bigl(-x^2/(2x_0^2)\bigr);$$

$H_0(z) = 1;\ H_1(z) = 2z;\ H_2(z) = 4z^2 - 2;\ H_3(z) = 8z^3 - 12z;\ H_4(z) = 16z^4 - 48x^2 + 12.$

Hydrogen atom:
normalised radial wave functions ($\int_0^\infty r^2 R^2 \mathrm{d}r = 1$), where $a_0 = \hbar c/(mc^2\alpha)$:

$$R_{10}(r) = 2a_0{}^{-3/2}\mathrm{e}^{-r/a_0},\ R_{20}(r) = (2a_0^3)^{-1/2}\left(1 - \tfrac{r}{2\,a_0}\right)\mathrm{e}^{-r/2a_0},\ R_{21}(r) = \bigl(24a_0^3\bigr)^{-1/2}\tfrac{r}{a_0}\mathrm{e}^{-r/2a_0}.$$

Expectation values for the state $|nlm\rangle$:

$$\langle r^2\rangle = \tfrac{1}{2}a_0^2 n^2\bigl(5n^2 + 1 - 3l(l+1)\bigr), \qquad \langle r\rangle = \tfrac{1}{2}a_0\bigl(3n^2 - l(l+1)\bigr), \qquad \langle r^{-1}\rangle = (n^2 a_0)^{-1},$$
$$\langle r^{-2}\rangle = \bigl((l+\tfrac{1}{2})n^3 a_0^2\bigr)^{-1}, \qquad \langle r^{-3}\rangle = \bigl(l(l+\tfrac{1}{2})(l+1)n^3 a_0^3\bigr)^{-1}.$$

Landé g-factor: $g_{jls} = 1 + \bigl(j(j+1) - l(l+1) + s(s+1)\bigr)/\bigl(2j(j+1)\bigr)$.

Spherical Harmonics:

$$Y_0^0(\theta,\phi) = \sqrt{\tfrac{1}{4\pi}}, \qquad Y_1^{\pm 1}(\theta,\phi) = \mp\sqrt{\tfrac{3}{8\pi}}\,\sin\theta\,\mathrm{e}^{\pm i\phi}, \qquad Y_1^0(\theta,\phi) = \sqrt{\tfrac{3}{4\pi}}\,\cos\theta.$$

Angular Momentum Operators:

$$\hat{J}_+ = \hat{J}_x + i\hat{J}_y, \qquad \hat{J}_- = \hat{J}_x - i\hat{J}_y, \qquad \hat{J}_\pm|j,m\rangle = \hbar\sqrt{j(j+1) - m(m\pm 1)}|j, m{\pm}1\rangle.$$

Pauli Matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Integrals: for positive integer n,

$$\int_0^\infty x^n \mathrm{e}^{-x/a}\mathrm{d}x = n!\,a^{n+1}; \qquad \int_0^\infty x^n \mathrm{e}^{-x^2/a^2}\mathrm{d}x = \begin{cases} (n-1)!!a^{n+1}/2^{(n+1)/2} & \text{if } n \text{ is odd,} \\ (n-1)!!\sqrt{\pi}a^{n+1}/2^{(n/2+1)} & \text{if } n \text{ is even.} \end{cases}$$

Time-independent perturbation theory:

$$|n^{(1)}\rangle = \sum_{m\neq n} \frac{\langle m^{(0)}|\hat{H}^{(1)}|n^{(0)}\rangle}{E_n^{(0)} - E_m^{(0)}}|m^{(0)}\rangle \qquad \text{and} \qquad E_n^{(2)} = \sum_{m\neq n} \frac{\Bigl|\langle m^{(0)}|\hat{H}^{(1)}|n^{(0)}\rangle\Bigr|^2}{E_n^{(0)} - E_m^{(0)}}.$$