PHYS30672 Physics with Theoretical Physics Core Unit
Dr. J. McGovern Physics Option Unit
Credit Rating: 10

Mathematical Methods for Physics (M)

Prerequisites PHYS20171, *PHYS20401, PHYS20672*
PHYS30201 is desirable but not essential.

Follow-up units Theoretical physics courses in 4th year

Classes 23 lectures in S6

Assessment 1 hour 30 minutes examination in May/June

Recommended texts

Arfken, G.B. Weber, H.J. *Mathematical Methods for Physicists*, 7th ed (Academic Press 2013)
Riley, K.F. Hobson, M. P. & Bence, S. J. *Mathematical Methods for Physics and Engineering* (CUP 2006)

Feedback

Feedback will be available on students' individual written solutions to examples sheets, which will be marked, and model answers will be issued.

Aims

The aim of this course is to achieve an understanding and appreciation, in as integrated a form as possible, of some mathematical techniques which are widely used in theoretical physics.

Learning outcomes

On completion successful students will be able to:

1. describe the basic properties of the eigenfunctions of Sturm-Liouville operators.
2. derive the eigenfunctions and eigenvalues of S-L operators in particular cases.
3. recognize when a Green's function solution is appropriate and construct the Green's function for some well known physical equations.
4. recognize and solve particular cases of Fredholm and Volterra integral equations.
5. solve a variational problem by constructing an appropriate functional, and solving the Euler-Lagrange equations.

6. solve problems related to the course material with the help of Mathematica.

Syllabus

1. **2nd Order ODEs and Sturm Liouville Theory** (7 lectures)

   Linear differential operators; Hermitian operators; eigenvectors and eigenvalues.

   Weight functions. Sturm-Liouville theory; Hermitian Sturm-Liouville operators. Spherical harmonics and Legendre's equation. The quantum oscillator and Hermite's equation. Orthogonal polynomials, recurrence relations. Series solutions. Laplace transforms.

2. **Green's functions** (5 lectures)

   Definition. Example: electrostatics. Construction of Green's functions: the eigenstate method; the continuity method. Quantum scattering in the time-independent approach; perturbation theory.

   Travelling waves. Example: electromagnetism. The Fourier transform method; retarded Green's functions and retarded potentials.

3. **Integral equations** (5 lectures)

   Classification: integral equations of the first and second kinds; Fredholm and Volterra equations.

   Simple cases: degenerate kernels; equations soluble by Fourier transform; problems reducible to a differential equation. Neumann series solution (perturbation theory); Fredholm series (if time). Eigenvalue problems; Hilbert-Schmidt theory.

4. **Calculus of variations** (6 lectures)

   Functionals: revision of stationary points and the Euler-Lagrange equation; the functional derivative; Fermat's principle; the brachistochrone. Generalization to more functions and variables. Constrained variational problems; Lagrange's undetermined multipliers. The isoperimetric problems. The catenary. The Rayleigh-Ritz method; application to quantum mechanics. The completeness theorem for Hermitian Sturm-Liouville operators.