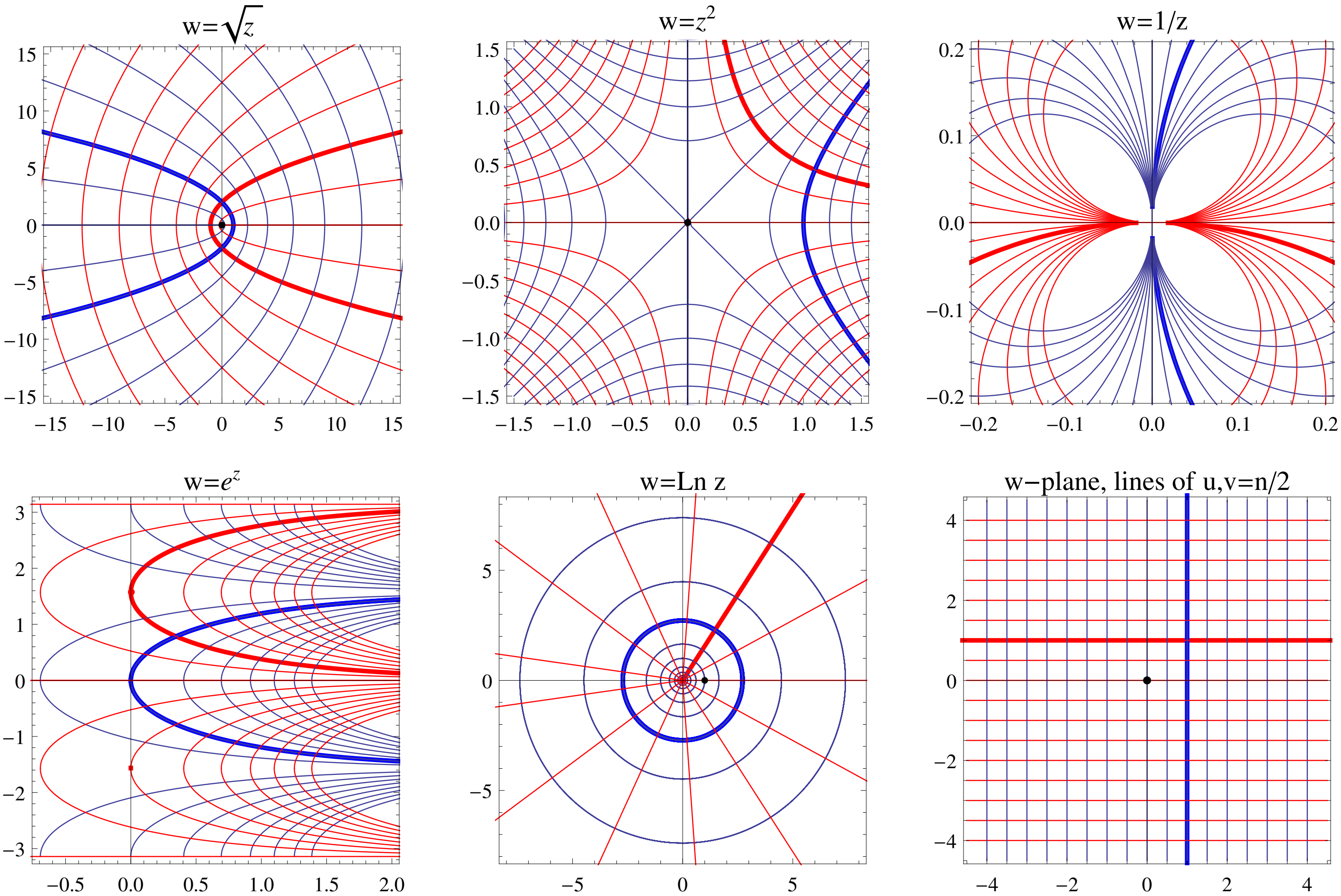
PHYS20672 Complex Variables and Integral Transforms: Handout 1



The first 5 pictures show lines of constant u (blue) or v (red), for a variety of mappings $w = f(z)$. Lines have been drawn for integer and half-integer values of u and v. The lines $u = 1$ and $w = 1$ are thicker than the others, and the point $w = 0$ is shown as a black dot.

For $w = \sqrt{z}$, $u = \pm a$ are same lines, as are $v = \pm b$.

For $w = 1/z$ the blank circular spaces would fill in if we included lines at higher and higher values of u and v. For this and for $w = \mathrm{e}^z$, the point $w = 0$ does not correspond to finite z (it is not in the range of the function).

For $w = \mathrm{e}^z$, with $u = \mathrm{e}^x \cos y$ and $v = \mathrm{e}^x \sin y$, values of y that differ by 2π correspond to the same values of u and v. So the pattern shown repeats, vertically shifted by $2n\pi$, to cover the whole plane. Note that for this plot the scales of the two axes are not the same, which is why the lines do not seem to cross at right-angles.

In the last picture, the w-plane is shown with a regular grid of lines $u =$constant (blue) and $v =$constant (red). These lines are the mappings of the z-plane curves onto the w-plane. Conversely a rectanglar grid on the z plane will map into the curves shown in the other plots under the inverse functions; thus the first graph shows the mapping of lines of constant x and y for the function $w = z^2$. The first two pictures in each line show inverse functions ($\sqrt{z}$ and z^2; e^z and $\ln(z)$). $1/z$ is its own inverse.