PHYS20352: Thermal and Statistical Physics

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Example Sheet 9

1. Consider a system of two independent particles occupying three (single-particle) states with energies: $\epsilon_1, \epsilon_2, \epsilon_3$.
(a) Assuming that the two particles are distinguishable. (i) List all the two-particle microstates by sketching the energy-level occupation diagrams. (ii) Calculate the two-particle partition function Z_2 at constant temperature T and show it can be written as
$$Z_2 = (Z_1)^2,$$
where Z_1 is the one-body partition function. (iii) Find the expression for the average energy $\langle E \rangle$ of the two-particle system and determine its value in the $T \to \infty$ limit.
(b) Now assuming the two particles are indistinguishable. Repeat part (a) and show in this case $Z_2 \neq (Z_1)^2$ and neither $(Z_1)^2/2!$. Show also that in the limit $T \to \infty$ the average energy $\langle E \rangle$ has the same value as that of part (a).

2. In the independent-particle approximation, calculate the partition function of the following many-body systems at constant temperature T and particle number N.

   (a) A perfect paramagnet of N spin-1 particles in an applied field. A single spin-1 particle in an applied field has only three energies: $-\epsilon, 0, \epsilon$.

   (b) A gas of N classical, relativistic particles (indistinguishable) moving in a volume V with a single-particle energy $\epsilon = cp$, where c is a constant and p is the magnitude of the 3-dimensional momentum. You may find the following integral useful
$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}, \quad a > 0.$$

3. (Challenge Question) One of the important integrals in undergraduate physics is the following Gaussian integral,
$$I = \int_{-\infty}^{\infty} e^{-ax^2} dx, \quad a > 0$$
which you may have seen in many places (certainly many times in this course). Here is your chance to solve it and obtain its exact value once for all. (Hint: It is useful to consider its square I^2.)