

PHYS20352: Thermal and Statistical Physics
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Example Sheet 8

1. (a) One joule of heat is added reversibly to 1 kg of water at a temperature of 300 K. What is the increase in entropy of the water? By what factor does the number of accessible microstates increase? [Hint: First calculate the increase in entropy using the isothermal expression, which is a very reasonable approximation here. Then use the Boltzmann law.]
(b) In the example of mixing two different gases (of 1 mole each) discussed in class, we obtained the entropy increase as $\Delta S = 2nR \ln 2$ with $n = 1$. Discuss how likely (or unlikely) the mixed gases might separate themselves into two halves of the volume respectively (i.e., each half only contains one kind of the gases).
2. Consider a paramagnet of N independent spin-1/2 particles in a magnetic field B . Its total energy is given by

$$E = (N - 2n)\mu_0 B,$$

where μ_0 is the magnetic moment of a single spin and n is the number of particles with spin up (hence, number of down spins is $(N - n)$).

- (a) What is the statistical weight Ω of the system at a given macrostate (E, B, N) ?
- (b) Using Stirling's approximation, $\ln m! \approx m \ln m - m$, for $m \gg 1$, show that the entropy of the paramagnet is given by

$$S = k_B [N \ln N - n \ln n - (N - n) \ln(N - n)].$$

- (c) Use definition of temperature to find $n = n(T)$ as a function of temperature and hence show that the internal energy equation is

$$E = -N\mu_0 B \tanh \frac{\mu B}{k_B T}.$$

- (d) Find the magnetization m as a function of temperature. Discuss its high- and low-temperature limits.