PHYS20352: Thermal and Statistical Physics

Yang.Xian@manchester.ac.uk, Room 7.14, Tel. 63692

Example Sheet 6

1. (a) From the fundamental thermodynamic relation for an open system, find expressions for the temperature T, pressure P and chemical potential μ in terms of partial derivatives of the entropy $S(E, V, N)$.

   (b) The entropy of a certain gas may be written in terms of its volume V and internal energy E as
$$S = \frac{4}{3} a^{1/4} V^{1/4} E^{3/4}$$
where a is a constant (i.e., independent of N, V and E). Show that the energy density of the gas may be written as $E/V = aT^4$, its pressure as $P = E/3V$, and its chemical potential $\mu = 0$. From your general knowledge of physics, can you recognize what kind of gas this is?

2. (a) The fundamental thermodynamic relation for a paramagnet in a magnetic field $\mathbf{B}$ is
$$dE = TdS - \mathbf{m} \cdot d\mathbf{B},$$
where $\mathbf{m}$ is the total magnetic moment. Derive the corresponding relation for an infinitesimal change in Helmholtz free energy F. Hence find entropy S in terms of F.

   (b) A paramagnet of N spin-1/2 particles is placed in a magnetic field B. Its Helmholtz free energy function F is found to be (as will be shown later in statistical mechanics part of the course)
$$F = -Nk_B T \ln\left(2\cosh\frac{\mu_0 B}{k_B T}\right),$$
where μ_0 is the magnetic moment of a spin. Find the entropy S of the paramagnet in terms of temperature. Show that in the high temperature limit, the entropy becomes
$$S \to k_B \ln \Omega, \quad \text{as} \quad T \to \infty,$$
with $\Omega = 2^N$. What is the significance of this Ω number?

3. (PC2352 Exam Question 2002) Write down the fundamental thermodynamic relation for an elastic string of tension Γ and length l. From the definition of the Helmholtz free energy, derive the following Maxwell relation:

$$\left(\frac{\partial S}{\partial l}\right)_T = -\left(\frac{\partial \Gamma}{\partial T}\right)_l .$$

The tension in an elastic string of unstretched length l_0 and temperature T is given by

$$\Gamma = \frac{RT}{l_0}\left[\frac{l}{l_0} - \left(\frac{l_0}{l}\right)^2\right].$$

By considering the entropy as a function of temperature and length, show that if the heat capacity at constant length is constant, $C_l = 3R$, the change in entropy between an initial state of length l_0 and temperature T_0 and another with length l and temperature T is

$$S - S_0 = 3R\ln\frac{T}{T_0} - R\left[\frac{1}{2}\left(\frac{l}{l_0}\right)^2 + \frac{l_0}{l} - \frac{3}{2}\right].$$

Hence find the final temperature if a string which is initially unstretched at 273 K is stretched slowly and adiabatically to twice its initial length.