PHYS20352: Thermal and Statistical Physics

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Example Sheet 3

1. A quantity, n moles, of an ideal gas is taken through the following three stages of a reversible cycle:

   - $a \to b$: a quasistatic isochoric (i.e., at constant volume, V_a) increase in pressure from P_a to P_b, with a corresponding increase in temperature, brought about by the absorption of an amount of heat Q_H from a series of external reservoirs ranging in temperature from T_a to T_b;
   - $b \to c$: a quasistatic adiabatic expansion from volume V_a to V_c, involving a corresponding drop in temperature from T_b to T_c; and finally
   - $c \to a$: a quasistatic isobaric (i.e., at constant pressure, P_a) decrease in volume from V_c to V_a with a corresponding decrease in temperature, brought about by rejection of an amount of heat Q_C to a series of external reservoirs ranging in temperature from T_c to T_a.

   (a) Draw the cycle on a standard P-V diagram.

   (b) Calculate the work done **by** the system, w, and the heat absorbed or rejected, during each of the three stages. [Note: You may find it useful to refer to the results of of Sheet 2 Q2 for the quasistatic adiabatic process.]

   (c) Verify that the total work w done by the system is equal to the net heat absorbed, namely, $(Q_H - Q_C)$, or more specifically,

   $$w = C_V(T_b - T_a) - C_P(T_c - T_a).$$

   [Note: You may find it useful to refer to the results of Sheet 2 Q1(a).]

   (d) Hence show the efficiency, $\eta \equiv w/Q_H$, of the system as a heat engine is given by

   $$\eta = 1 - \frac{\gamma(1-r)}{r(1-r^\gamma)}\frac{T_a}{T_b},$$

   where $\gamma \equiv C_P/C_V$, and $r = V_a/V_c$ is the inverse of the compression ratio.

2. (a) Use the first law of thermodynamics for a general gas (i.e., not necessarily an ideal gas) to derive the two following equations:

   $$đQ = C_V dT + \left[\left(\frac{\partial E}{\partial V}\right)_T + P\right] dV,$$

where C_V is the heat capacity at constant volume and

$$C_P = C_V + \left[\left(\frac{\partial E}{\partial V} \right)_T + P \right] \left(\frac{\partial V}{\partial T} \right)_P ,$$

where C_P is the heat capacity at constant pressure.

(b) Now consider one mole of a van der Waals gas, obeying the equation of state

$$\left(P + \frac{a}{V^2} \right) (V - b) = RT \, .$$

Its internal energy is given by

$$E = cT - \frac{a}{V} \, ,$$

where a, b, and c are constants, and R is the gas constant. Hence show that the molar heat capacities at constant volume and constant pressure are given respectively, by

$$C_V = c \, , \quad C_P = c + \frac{R}{1 - 2a(V - b)^2/(RTV^3)} \, .$$