PHYS20352: Thermal and Statistical Physics

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Example Sheet 2

[Note: *This 2nd sheet involves some typical derivations in thermodynamics, parts of which may look unfamiliar to you because of more than one variables. Please refer to Lecture Notes and recommended books for help, do as much as you can, and discuss the difficult parts with your tutor.*]

1. The equation of state for n moles of an ideal gas contained in a volume V, and at pressure P and temperature T, is given by

$$PV = nRT\,.$$

(a) Using the fact that the internal energy E of an ideal gas is state function of the temperature only, $E = E(T)$, show that from the first law of thermodynamics that the difference between the heat capacities at constant pressure and at constant volume of n moles of an ideal gas is given by

$$C_P - C_V = nR\,.$$

(b) Hence show that the heat transferred in an infinitesimal quasistatic process involving n moles of an ideal gas is given by

$$đQ = \frac{1}{nR}(C_V V dP + C_P P dV)\,.$$

(c) Hence show that for a quasistatic adiabatic process involving an ideal gas, for which the heat capacities are constants (i.e., independent of temperature),

$$PV^\gamma = K\,,$$

where K is a constant, and $\gamma \equiv C_P/C_V$.

2. As we proved in Question 1(c) above, during a quasistatic adiabatic compression of an ideal gas, the pressure P and volume V are related by the equation

$$PV^\gamma = K\,.$$

(a) Show that the work done on the gas during a quasistatic adiabatic compression from a state (P_i, V_i, T_i) to state (P_f, V_f, T_f) is given by

$$W = \frac{P_f V_f - P_i V_i}{\gamma - 1} = \frac{P_i V_i[(V_i/V_f)^{\gamma-1} - 1]}{\gamma - 1}\,.$$

Note: This result may be compared with those of Sheet 1 Q2(a) and (b) of last week.

(b) By making use of the result in Question 1(a) above, show that this equation for W can also be written as

$$W = C_V(T_f - T_i)\,.$$

Show how this latter expression also follows directly from conservation of energy and the fact that C_V is constant (i.e., independent of temperature) for an ideal gas.

(c) If the volume of the gas is halved during the compression, show that

$$T_f = 2^{\gamma-1}T_i\,.$$